# Calibration Technique for Superfluid $^4$He Weak-Link Cells Based on the Fountain Effect


E. Hoskinson and R. E. Packard

*Department of Physics, University of California, Berkeley, CA 94720, USA*



**Abstract.** Studies of superfluid $^4$He weak-links require calibration constants which permit the determination of the pressure and temperature differences which drive Josephson oscillations. We describe a technique for calibrating $^4$He weak-link cells in which a heater is used to induce fountain pressures detected by the deflection of a diaphragm. The technique determines the diaphragm spring constant, the inner cell volume, and the thermal conductance of the inner cells walls. This information is used to convert the measured deflection of the diaphragm into the total chemical potential difference across the weak link.



**Keywords:** weak-link, superfluid, calibration
**PACS:** 67.40.Rp, 67.40.Hf


Low frequency Helmholtz resonator-type cells for the study of superfluid helium flow through small apertures have been used in a variety of experiments. These cells utilize a flexible diaphragm as a piston to drive and detect fluid flow through the apertures. Flow can be driven electrostatically by application of a voltage between the metalized diaphragm and a nearby electrode. It can also be driven via the fountain effect with a heater. Displacement of the diaphragm, indicating both fluid flow (which directly displaces the diaphragm), and pressure (because the diaphragm has a spring constant), can be measured capacitively or with a SQUID-based superconducting displacement transducer. We have used such a cell to study superfluid $^4$He quantum oscillations in an array of submicron sized apertures [1, 2]. Here we describe the method we use for measuring several dynamical quantities of the cell, and present data from such measurements.

A schematic of our cell is shown in figure 1. It consists of a cylindrical container of inner diameter 8 mm and height 0.6 mm, with walls and bottom constructed out of aluminum 7075. The top boundary is a 8 $\mu$m thick flexible Kapton diaphragm, with a 400 nm layer of superconducting lead evaporated onto the top surface as part of the diaphragm displacement transducer. The array, mounted on the bottom plate, consists of 4225 nominally 60 nm diameter apertures in a square array with 3 $\mu$m spacing, in a 50 nm thick 200 $\mu$m $\times$ 200 $\mu$m silicon-nitride membrane. This membrane is supported on a 3 mm $\times$ 3 mm, 0.5 mm thick silicon chip, glued into the bottom plate using Stycast 2850 FT with catalyst 24LV. The heater is a 54 m$\Omega$ length of CuNi wire, flattened and roughened to increase surface area. Unclad 50 $\mu$m superconducting NbTi wire are used for the heater leads to minimize thermal conduction along them and ensure all power delivered is dissipated inside the inner volume.

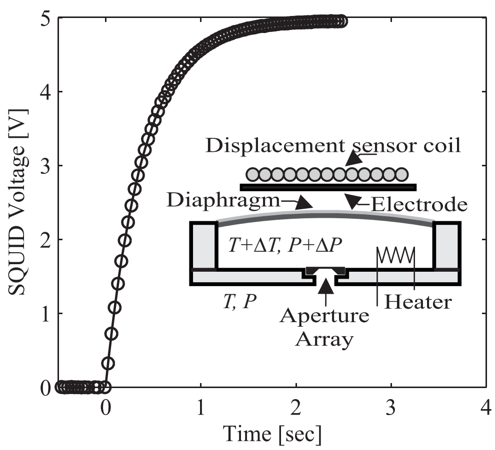


**FIGURE 1.** Displacement sensor voltage output versus time and a schematic of the experimental cell. The heater is turned on at time t = 0 sec. The temperature across the aperture array relaxes to a steady state value $\Delta T_f$, determined mainly by thermal conduction through the inner cell walls. The diaphragm relaxes to a new position corresponding to the fountain pressure $\Delta P_f = \rho s \Delta T_f$.

Our method is based on measurement of heater driven relaxation transients. One such transient is shown in figure 1. When power $W$ is suddenly applied to the heater inside the inner cell, a temperature difference $\Delta T(t)$ across the aperture array grows, driving superfluid current $I_s$ into the inner cell (the fountain effect), causing the diaphragm to bulge and the pressure $\Delta P(t)$ to rise.

The diaphragm displacement transducer output signal is a voltage, $\Delta U$, proportional to displacement, $x = \alpha \Delta U$. Displacement is proportional to $\Delta P$ : $kx = \Delta P A$, where $k$ is the diaphragm spring constant and $A$ is its area. Thus $\Delta P = k\alpha\Delta U/A \equiv \gamma\Delta U$. The constant $\gamma$ is de-


CP850, *Low Temperature Physics: 24th International Conference on Low Temperature Physics;*
edited by Y. Takano, S. P. Hershfield, S. O. Hill, P. J. Hirschfeld, and A. M. Goldman

termined from the Josephson frequency $f_j = \Delta\mu/h$ measured at the beginning of a pressure driven transient (excited electrostatically) where $\Delta T = 0$ [1]. Here $\Delta\mu = m_4(\Delta P/\rho - s\Delta T)$ is the chemical potential difference across the aperture array, $h$ is Plank's constant, $m_4$ is the $^4$He atomic mass, $\rho$ is the fluid mass density and $s$ is the fluid entropy per unit mass.

The total current $I_t = I_s + I_n$ into the inner cell, where $I_s$ and $I_n$ are the superfluid and normal components, is related to the diaphragm motion and $\Delta P$ through: $I_t = \rho A \partial x/\partial t = \rho(A^2/k)\partial\Delta P/\partial t$.

The evolution of $\Delta T(t)$ is determined by a balance of heat flows [2]:

$$C_p \frac{d\Delta T}{dt} = -sT\left(I_s - \frac{\rho_s}{\rho_n} I_n\right) - \frac{\Delta T}{R} + W. \qquad (1)$$

Here $\rho_s$ and $\rho_n$ are the superfluid and normal fluid densities, $R$ is the thermal resistance between the $^4$He inside the inner cell and the $^4$He outside it, and $C_p = c_p V$, where $c_p$ is the heat capacity per unit volume of $^4$He and $V$ is the volume of the inner cell.

The size of our apertures is such that the viscous normal flow, while small in comparison with the superflow, is not entirely negligible. Viscous normal flow follows

$$I_n = -\frac{\rho_n \beta}{\eta}\left(\frac{\rho_n}{\rho}\Delta P + \rho_s s \Delta T\right), \qquad (2)$$

where $\beta$ is a geometrical factor and $\eta$ is the viscosity. We measure $\beta$ directly from the flow response to an electrostatically induced $\Delta P$ just above the superfluid transition temperature $T_\lambda$.

If the heater power applied is sufficiently small, no Josephson oscillations are excited and $\Delta\mu = 0$ is maintained throughout the transient. In this case $\Delta P$ will relax exponentially to a steady state fountain pressure

$$\Delta P_f = \frac{\rho s R W}{(1 + \rho^2 s^2 T R \beta/\eta)}, \qquad (3)$$

with time constant

$$\tau_f = \frac{\Delta P_f}{\rho s W}\left(C_p + s^2\rho^2 T \frac{A^2}{k}\right). \qquad (4)$$

The constants $A^2/k$ and $V$ (recall $C_p = c_p V$) are determined from a fit to $\tau_f/\Delta P_f$ as a function of $T$ (with $\tau_f$ and $\Delta P_f$ determined from transients such as the one shown in fig. 1). Alternatively, if $V$ is already known, but the pressure calibration constant $\gamma$ is not, the fit can determine $A^2/k$ and $\gamma$. Published values are used for $s$, $\rho_s$, $\rho_n$, $\eta$, and $c_p$ [3]. For the data shown in fig. 2, the parameters values were determined to be, with estimated uncertainty in the last digit specified in brackets, $\gamma = 0.0313(6)$ Pa/V, $k/A^2 = 1.88(4)\times 10^{12}$ N/m$^5$, $V = 2.45(5)\times 10^{-8}$ m$^3$, $\beta = 4.8(3)\times 10^{-20}$ m$^3$.

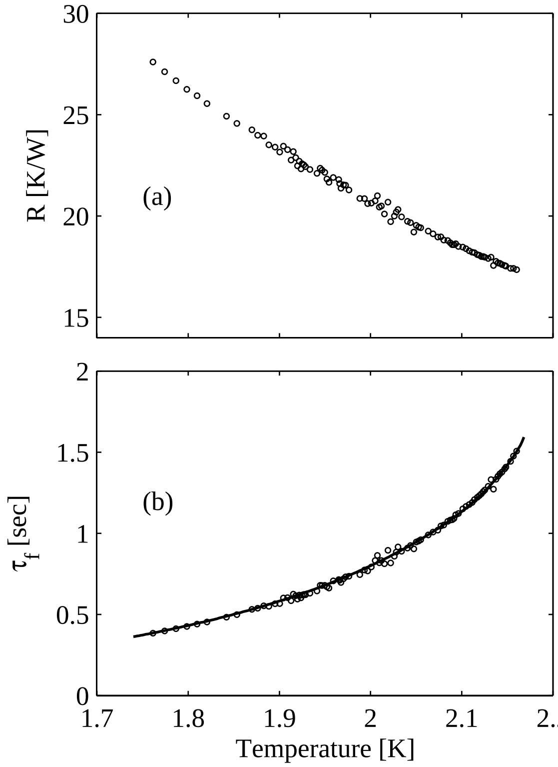


**FIGURE 2.** (a) Thermal resistance $R$ from the measured fountain pressure $\Delta P_f$ versus temperature. $R$ is likely determined mainly by conduction through the thin, large area diaphragm. Heat transport by normal flow through the apertures is taken into account separately, and is small compared to thermal conduction. (b) Fountain transient time constant $\tau_f$ versus temperature, data (circles), and fit (line), from which $A^2/k$ and $V$ are determined.

With the calibration constants determined using this method, eq. 1 can be numerically integrated, giving $\Delta T(t)$ and thus $\Delta\mu(t)$ for any measured $I_t(t)$, $\Delta P(t)$. The quantum mechanical phase $\Delta\phi$ across the aperture array can be determined by integration of the Josephson-Anderson phase evolution equation $d\Delta\phi/dt = -\Delta\mu/\hbar$.

This work is supported in part by the NSF-DMR and NASA.

## REFERENCES


1. E. Hoskinson, R. E. Packard, and T. M. Haard, *Nature* **433**, 376 (2005).
2. E. Hoskinson and R. E. Packard, *Phys. Rev. Lett.* **94**, 155303 (2005).
3. R. J. Donnelly and C. F. Barenghi, *J. Phys. Chem. Ref. Data* **27**, 1217 (1998).