therefore difficult to make the $k=0$ extrapolation in Eq. (2) when $\omega \to 0$.

This molecular-dynamics calculation shows that the vaf of a dense gas at a near-critical temperature behaves asymptotically like $\alpha t^{-3/2}$; the value of the coefficient $\alpha$ can be deduced from the transport coefficients $\nu$ and $D$. These results confirm those obtained by Alder and Wainwright[1] and by Wood[9] for systems of 500 and 4000 hard spheres.

The authors are particularly grateful to L. Verlet and J. J. Weis for their help in realizing this work.

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# Photographs of Quantized Vortex Lines in Rotating He II*

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(Received 28 May 1974)

The spatial positions of discrete quantized vortex lines in rotating superfluid helium have been directly visualized by a photographic technique. The positions of the lines in the apparatus do not form a regular array.

The unique properties of He II have been associated with the existence of a macroscopic wave function which determines the behavior of the superfluid component. From this idea Onsager[1] and Feynman[2] predicted that He II should exhibit vorticity with circulation quantized in units of $h/m$, where $h$ is Planck's constant and $m$ is the mass of the helium atom. For almost two decades physicists have been exploring theoretically and experimentally the phenomena associated with these vortices.[3] A wealth of convincing information is available which supports the existence of vortices, the most direct experiments being those which proved that circulation in He II is quantized[4,5] and that He II comes into rotation in a series of quantum steps.[6] There has still been one experiment which has enticed workers for some time: actually to make directly visible the discrete vortex lines in the rotating He II. (An analogous experiment has been done to visualize fluxoids in a superconductor.[7]) This Letter describes the first successful experiment which records the positions of the vortex lines in helium. We point out that according to current ideas the vortex core is a node in the macroscopic wave function. This is one of the only measurements we know which directly measure the positions of the nodes of a wave function.

The method we employ is conceptually simple.[8] Vortex lines should appear in a container of He II rotating at angular velocity $\omega$, with a predicted density[2] of $2\omega m/h$ lines/cm$^2$. Electrons formed near a radioactive source are injected into the rotating He II. The electrons form bubbles (radius ~ 16 Å) which become trapped on vortex lines in a Bernoulli trapping potential.[9] The lines are charged for about 10 sec, after which an axial electric field is applied which pulls the ions through the liquid meniscus. Once free of the liquid, the electrons are accelerated and impinge on a phosphor screen where they produce a flash of light, thus marking the position of the line where it meets the free surface.

There are numerous complications which conspire to defeat the simplicity of this method. First of all, the experiment must be done at temperatures below 0.6 K so that the helium vapor pressure is low enough to allow the use of electron optics. Second is the empirical fact that electrodes in the space above the liquid will support a potential difference of only about 500 V without an electric discharge occurring. This limits the electron accelerating energy and consequently the light output. A more fundamental problem is the question of stability of the vortex lines. Although calculations show that in equilibrium the vortices should form a triangular lattice,[10,11] at rest in the rotating frame, it is not known how large a disturbance would be necessary to destroy the regularity of the array. Several authors have presented the opinion that the vortices would not exhibit a rigid lattice in a real situation.[10,12]

The basic experimental configuration is shown in Fig. 1. Electrons, formed near a 13-mCi tritium source, are manipulated by applied potentials so that they become trapped on the vortex lines. When the maximum charge is stored, the source is switched off and the charge is extracted and accelerated into a ZnO:Zn phosphor. An axial magnetic field of 3 kG prevents defocusing of the electrons. The phosphor is deposited onto a fiber optics disc which is covered with a transparent conductive coating.[13] The disc butts against

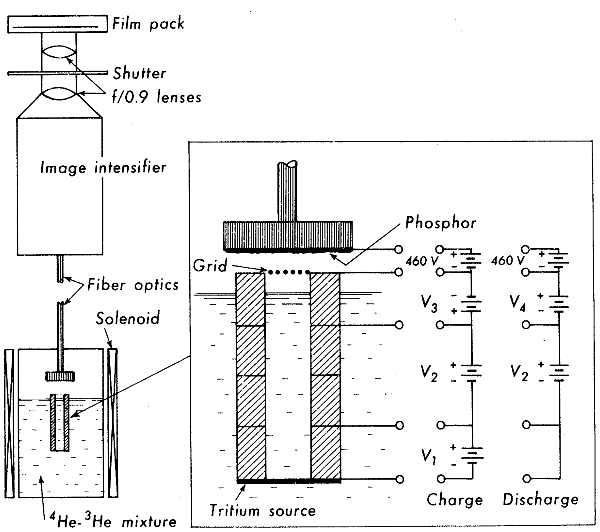


FIG. 1. Cross-sectional sketch of the electron optics used to photograph vortex lines. The chamber containing the cell is cooled by a dilution refrigerator. The inset shows the electrodes used to charge and discharge the vortex lines. In the 0.8% mixture the bias potentials used were $V_1=45$, $V_2=90$, $V_3=8$, and $V_4=45$ V. The "bucket" is 3 cm long and consists of a 3.8-mm-diam hole drilled in four carbon resistors (Ref. 6). The voltage gradient along the resistive walls produces the required electric fields.

a 3.2-mm-diam image conduit which conveys the light to room temperature.[14] The light signal is quite weak (of the order of 100 photons per vortex line at the room-temperature end) and is amplified with a three-stage image intensifier.[15] The intensifier output is imaged 1:1 onto high-speed recording film[16] through optics with an effective aperture of $f/0.9$. The overall system, including the electron optics, has a resolution of approximately 10 lines/mm at 30% modulation and a sensitivity sufficient to record an image due to several hundred electrons hitting the phosphor at one point within a 30-msec period.

This system was initially part of a $^3$He refrigerator which allowed the cell to be cooled to 0.3 K.[17] The entire apparatus, including the camera system and refrigerator, is smoothly rotated on an air bearing. Extensive resolution tests showed that this apparatus possessed the sensitivity and resolution to photograph individual vortices. However, the actual photographs using rotating He II showed no discernible detail; i.e., the image of the light, produced by the electrons emitted by the vortices, was a complete blur. We conclude that at temperatures near 0.3 K the vortex array in $^4$He is not stable on the time scale of the exposure time (~30 msec). This is not surprising because at low temperatures there is a negligible amount of normal fluid to damp any nonequilibrium motion of the lines. The instability of the vortex array can also be seen in measurements of trapped-electron lifetimes.[18]

In order to stabilize the vortices we decied to add $^3$He to the superfluid. At low temperatures the $^3$He acts as a fixed amount of normal fluid and thus provides a damping mechanism for nonequilibrium motion of the vortices. Since the addition of $^3$He raises the vapor pressure of the fluid[19] it was necessary to perform the experiment at temperatures lower than 0.3 K. Thus we assembled a rotating dilution refrigerator[20] which provides temperatures below 0.1 K. The inset in Fig. 1 shows the cell used for the mixtures.

The addition of $^3$He does indeed act to stabilize the vortices and we finally see, for the first time, the image of the vortex-line positions. Figure 2 shows three typical photographs taken with a $^3$He concentration of 0.8%. The concentrated white spots mark the location of the vortex lines. Based on approximately 50 such pictures, several comments can be made.

(1) A stable triangular lattice is not apparent in the photographs.

(2) Even with 0.8% $^3$He there is still some mo-

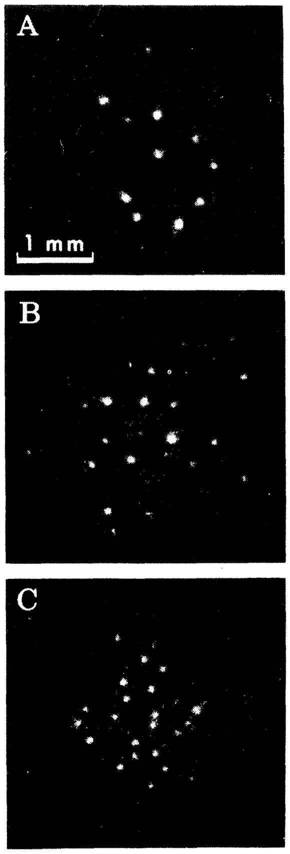


FIG. 2. Photographs of discrete quantized vortex lines in He II rotating at (a) $\omega = 0.25\ \text{sec}^{-1}$, (b) $\omega = 0.5\ \text{sec}^{-1}$, (c) $\omega = 1.0\ \text{sec}^{-1}$. The temperature was approximately 75 mK, exposure time 60 msec. The distinct bright spots mark the positions of the lines where they meet the free surface. Because of a slight misalignment of the fiber optics the field of view in these pictures is only about one half of the bucket. The center of the picture does not coincide with the center of the bucket.

tion of the vortices, as deduced from the fact that consecutive photographs, taken less than 1 min apart, show no similarity in the distribution of vortex lines. Whether the random motion of the vortices is intrinsic to the rotating He II or is caused by mechanical disturbances in the apparatus is difficult to determine. The rotating refrigerator is supported on an air bearing and we have taken precautions to eliminate most sources of vibration. Nevertheless, only future tests will shed light on this question

(3) Well above the predicted critical velocity for the first line the pictures show that the number of lines increases linearly with angular velocity but there are less than $2\omega m/h$ lines per unit area (20 lines/mm$^2$ for $\omega = 1\ \text{sec}^{-1}$). Our method records only those vortices which have not migrated to the walls during the 10-sec charging period. In addition the line must terminate at the free surface rather than the bucket's wall. We as yet do not have enough data to make any quantitative statements on the fraction of lines which might not be displayed in the pictures. In the pictures taken in the 0.8% mixture there seem to be about one half the number of lines predicted from theory.

(4) When many lines are visible the nearest-neighbor spacing is approximately $(h/2\omega m)^{1/2}$ as expected from Feynman's argument.

(5) We have thus far taken pictures at $^3$He concentrations of 0.1, 0.4, and 0.8%. It seems that the lines become more stable at the higher concentrations. Future observations will explore this phenomenon.

An obvious question to ask is what is the evidence that the dots in the photographs do indeed mark the positions of the vortex lines? We have performed several tests to check this point.

(i) Using a tunnel-cathode emitter and a pinhole grid to simulate the low-energy electrons leaving the vortex lines, we found that with the magnetic focusing field on, the charge was accurately imaged onto the phosphor. With the field off, a uniform blur was obtained. In the actual helium experiment if the focusing field is turned off the dots in the picture disappear, producing a uniform blur.

(ii) If the electric field in front of the radioactive source is turned off, the photographs show no signal with a typical exposure time (60 msec). A longer exposure (500 msec) shows a uniform background, presumably caused by the ultraviolet light produced in the liquid helium near the radioactive source.

Now that we have demonstrated the feasibility of this vortex-line imaging system there are numerous interesting questions whose answers can be sought. The most useful extension of this technique would be the addition of a TV system to provide a real-time image of the lines. Then one could hope to observe the detailed motion of the lines in the vessel and thereby determine if classical hydrodynamics accurately determines the line motion. In addition one could see where the lines are created.

Our immediate plans are to learn about the stability conditions of the vortex array by photographing the vortices in various geometries and state preparations, and different $^3$He concentrations.

In summary, we have produced direct visual evidence that individual, discrete vortex lines exist as predicted by Feynman and Onsager. It appears that under our experimental conditions no stable vortex array is observed; rather the rotating He II exists in a dynamic state involving complex motion of vortex lines.

We are grateful to T. M. Sanders who suggested the electron trapping technique and provided many stimulating discussions. We also thank K. DeConde and F. Reif who gave frequent helpful comments.

*Work supported by the National Science Foundation.

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# Mass-Fluctuation Waves in Solid $^3$He-$^4$He Mixtures*

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We give an explanation of the data of Richards, Pope, and Widom in terms of the incoherent tunneling of strongly interacting particles.

The concept of "impuritons" or "mass-fluctuation waves" was invented independently by Andreev and Lifshitz[1] and Guyer and Zane.[2] The early survey experiments by Myoshi, Cotts, Greenberg, and Richardson[3] suggested that dilute solid mixtures of $^3$He in $^4$He are a suitable system in which to seek evidence for these excitations.[4] Subsequently Greenberg, Thomlinson, and Richardson[5,6] reported $T_1$ and $T_2$ measurements that showed compelling evidence for $^3$He tunneling in $^4$He. Richards, Pope, and Widom (RPW)[7,8] have undertaken an extensive set of measurements on dilute $^3$He in $^4$He mixtures designed to find evidence for the coherent motion of the impurity, i.e., for the existence of $^3$He "impuritons." These measurements include diffusion data in addition to $T_1$ and $T_2$ data. RPW have argued that a diffusion measurement gives a clear signature for coherent motion. Recently Grigor'ev and co-workers[9] have verified the dif-

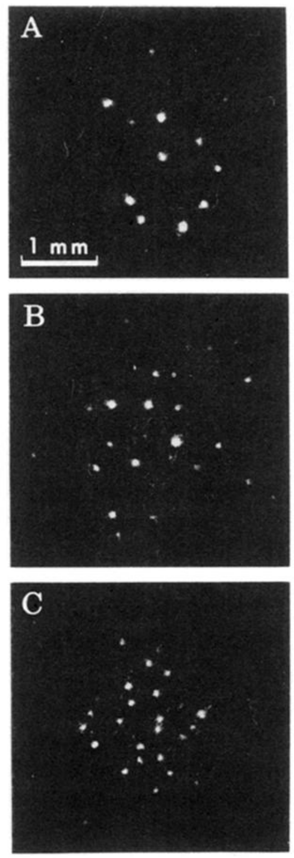


FIG. 2. Photographs of discrete quantized vortex lines in He II rotating at (a) $\omega = 0.25\ \mathrm{sec}^{-1}$, (b) $\omega = 0.5\ \mathrm{sec}^{-1}$, (c) $\omega = 1.0\ \mathrm{sec}^{-1}$. The temperature was approximately 75 mK, exposure time 60 msec. The distinct bright spots mark the positions of the lines where they meet the free surface. Because of a slight misalignment of the fiber optics the field of view in these pictures is only about one half of the bucket. The center of the picture does not coincide with the center of the bucket.