Quantized Phase Slippage in Superfluid ^{4}He

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We have studied dissipative phase-slip events in the oscillatory superflow of ^{4}He through two different orifices of submicron size. The dissipation events are found to be caused by the phase difference across the orifice changing by $2\pi n$, where the slip integer n depends on the orifice geometry, flow direction, and temperature. The temperature dependence of the phase-slip critical velocity indicates that surface microstructure determines the magnitude of relevant energy barriers.

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Many recent experiments have focused attention on the processes leading to dissipation in the flow of superfluid through small orifices [1]. Anderson [2] predicted that dissipation can occur when a quantized vortex passes across the orifice, removing energy from the potential flow field. If the vortex crosses all the flow lines passing through the orifice, the quantum phase difference across the hole changes by 2π. Experiments by Avenel and Varoquaux (AV) demonstrated that individual vortex dissipation events, each characterized by a phase slip of 2π, can be observed under appropriate circumstances [3–5]. Their work has raised questions directed at understanding the mechanisms underlying these events.

We report here the results of a systematic investigation of the phase-slip phenomena in two micro-orifices. The data reveal new features which considerably expand our understanding of the superfluid dissipation processes.

Our apparatus is schematically quite simple. A superfluid-filled cell is partitioned into two halves by a wall containing a flexible, niobium-coated Mylar diaphragm and a single submicron orifice. A metal electrode in the shape of a flat disk is fixed in place near one side of the diaphragm, and serves as an electrostatic driver that can apply controllable forces to the diaphragm. On the opposite side of the diaphragm is placed a planar superconducting coil which forms the input of a dc-SQUID-based displacement sensor [6] capable of resolving diaphragm motions as small as 10^{-13} m in 1 s when a current of 0.15 A is trapped in the planar coil.

Superflow is driven through the orifice, from one side of the cell to the other, by applying appropriate voltages to the metal electrode. The kinetic inductance of the flow path, determined by the orifice geometry, couples to the restoring force of the diaphragm to produce a Helmholtz resonance with a frequency of 1 to 2 Hz. Since the orifice provides the only flow path between the two sides, the measured volume displacement rate of the diaphragm is equal to the rate of volume flow through the micro-orifice under study [7]. In this respect, our apparatus is different from that used by AV, which had a second, more open flow channel in parallel with the micro-orifice connecting the two sides [1,3–5]. The simpler topology of our apparatus eliminates the need to account for the effect of hydrodynamic circulation that can be trapped in the loop threading the micro-orifice and the second channel, and minimizes the possibility of detecting phase-slip events anywhere except in the orifice under study.

The orifices are fabricated in 0.1-μm-thick membranes of silicon nitride using electron-beam lithography [8]. The data presented herein are based on measurements using two different orifices. Orifice 1 is nominally a square hole of dimensions 0.35 μm$\times$0.35 μm. Orifice 2 is a rectangular slit of size 4.25 μm$\times$0.35 μm. Scanning electron microscopy images show orifices whose walls are smooth on a scale of 0.01 μm, while the corners of the opening are rounded on a scale of 0.1 μm.

The apparatus, which can be cooled as low as 0.26 K, is vibration isolated to a degree that random vibrations excite the Helmholtz mode to an amplitude which is $\frac{1}{10}$ as large as the change seen in a 2π phase slip.

Figure 1 shows the typical time evolution of the oscillator's amplitude while being driven at its Helmholtz resonance by the application of a small electrical drive. Each point represents the oscillation amplitude of a half cycle at the instant of maximum diaphragm displacement. The sudden drops in the resonator amplitude signify dissipative phase-slip events of the type observed by AV. By recording the actual, unfiltered wave form of the

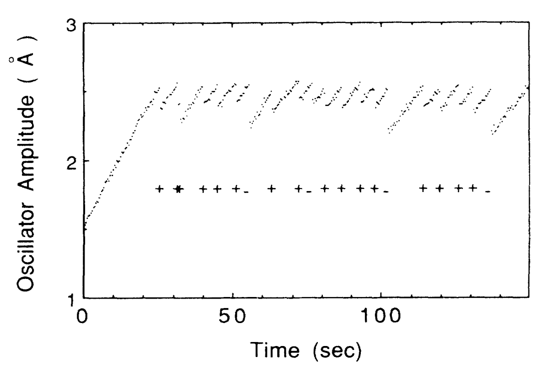

FIG. 1. Phase slips in the time evolution of resonator amplitude at T=0.26 K for orifice 1. Resonator Q is $\sim 10^4$. The plus or minus sign shown below each jump gives the direction of flow through the orifice at the time of the phase slip.

SQUID output on a storage scope, we find that the dissipative jump occurs when the diaphragm passes through the zero of displacement, i.e., at the maximum velocity. Furthermore, the event gets completed in a time scale shorter than the experimentally resolvable time of 1 ms.

It can be seen from Fig. 1 that not all the dissipation events are of the same magnitude. Figure 2 shows histograms, for both flow directions, of phase-slip sizes recorded at a fixed temperature for orifice 1. We find that the size of the resonator amplitude loss in a phase slip is quantized, being an integer multiple of $(1.55\pm0.01)\times10^{-11}$ m, which is the location of the first peak on each histogram. The question then arises: Do the events associated with the first peak correspond to phase slips of 2π?

In the model introduced by Anderson, a 2π phase slip will decrease the energy in potential flow by an amount $\Delta E_{2\pi}=\kappa I_c$. Here κ is the circulation quantum $(9.98\times10^{-8}$ m^2/s) and I_c the mass current at which the event occurs. In order to compare the observed phase-slip energy loss $\Delta E_{\rm obs}$ to that predicted by Anderson, one may compare the ratio $R=\Delta E_{\rm obs}/I_c$ with the circulation quantum κ. In terms of experimental quantities that can be measured accurately [9],

$$R=\varepsilon\Delta V_{\rm slip}/4\pi\alpha\rho_s f d^2\,, \qquad (1)$$

where ε is the permittivity of liquid helium, ρ_s the superfluid density, f the resonance frequency of the oscillator, and d the low-temperature value of the diaphragm-to-electrode gap. $\Delta V_{\rm slip}$ is the decrease in the amplitude of the SQUID output voltage during the phase slip, and is known (from averaging 400 events) to within $\pm0.5\%$. The parameter α, which is proportional to the mechanical compliance of the diaphragm, is obtained to better than 0.5% from measuring the change of SQUID output $V_{\rm sq}$ as a function of the dc voltage V applied between the metal electrode and diaphragm. That function

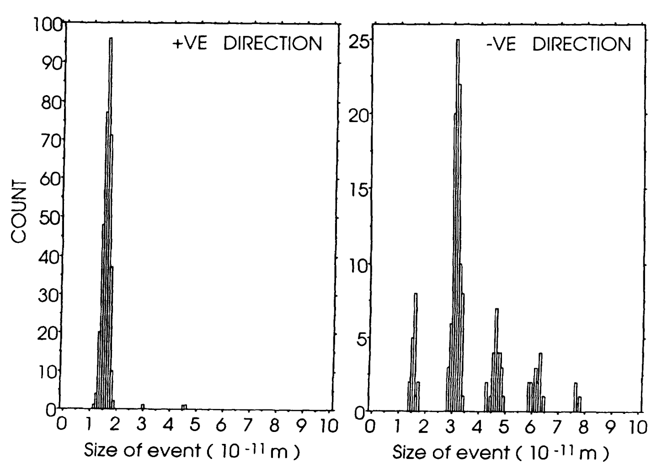

FIG. 2. Distribution of sizes of phase slip events for orifice 1, in terms of the associated amplitude loss from the resonator. Separate histograms are shown for the two flow directions. $T=0.27$ K.

is found to be well described by the form $V_{\rm sq}=\alpha V^2$.

The gap d is found, from capacitance measurements when filling the cell with superfluid helium, to be 139 μm with an estimated systematic error that could be as high as 3%.

With these experimental calibrations, we determine the ratio R for the smallest-sized phase slips to be $(9.6\pm0.7)\times10^{-8}$ m^2/s for orifice 1 and $(9.8\pm0.7)\times10^{-8}$ m^2/s for orifice 2, in good agreement with the known value of h/m_4. This gives us confidence that the smallest slips correspond to single 2π vortex events.

Since the size of the observed phase slips is always a multiple of the 2π unit, one can associate with any phase slip an integer n, the number of vortices involved in the event, so that the slip changes the quantum phase difference across the orifice by $2\pi n$. From Fig. 2 it can be seen that, whereas the slip quantum number n is almost exclusively 1 for flow in one direction (arbitrarily called the positive or + direction), it is often greater than 1 for flow in the opposite (−) direction. This characteristic asymmetry is even more pronounced in the case of orifice 2. For that aperture the histograms analogous to Fig. 2 show an abundance of phase slips with n as high as 100 for the flow direction characterized by multiple slips, whereas the slips in the opposite direction are almost always $n=1$, and never exceed $n=2$.

The data in Fig. 2 differ from those of Refs. [3-5] by the occurrence of so many multiply quantized slips in one direction. The studies of Refs. [3-5] employed a nominally rectangular aperture of dimensions 5 μm$\times$0.3 μm with a highly irregular perimeter as seen in a TEM picture [4], and almost all the phase slips were characterized by $n=1$. The difference between those data and ours, as well as the difference between the histograms of slip sizes for our two specimen apertures, indicates the likelihood that the slip number n of an individual dissipation event depends on the detailed geometry of the orifice.

We find the observed critical velocity for the triggering of phase-slip events to be identical in both directions (see Fig. 4). Thus it would appear that the nucleation mechanism for the phase-slip vortex is independent of flow direction. This might be expected since, before vorticity appears, the potential flow field is determined by the geometry but does not depend on direction (except for the sign). However, once the critical velocity is reached and a vortex nucleated, the complex evolution of quantized turbulence could be influenced by the combination of boundary and flow direction. This may be similar to the case of classical fluids in which the evolution of turbulence depends on both direction and boundary shape.

We suspect that $n>1$ phase slips occur when a vortex filament moving across the aperture bows out, twists, reconnects, and pinches off [10], thus ejecting a single vortex ring in addition to the remainder element, both of which ultimately traverse all the flow lines linked to the orifice. Such a process will produce an $n=2$ phase slip. If the bowing, twisting, and pinching-off process occurs

$n-1$ times, the phase slip will be of size $2\pi n$.

Some support for this picture is to be found in Fig. 3 which shows for orifice 1 the temperature dependence of the average value $\langle n \rangle$ of the slip quantum number, for the $-ve$ flow direction. As the temperature increases, $\langle n \rangle$ falls until it reaches the lowest possible value, $n=1$. The explanation for this behavior might lie in mutual friction which increases with temperature and can provide a mechanism which damps unstable vortex motion thus inhibiting the twisting reconnection events conjectured above.

We have looked into the possible connection between extraneous vibrations and the distribution of phase-slip sizes. While high levels of vibration or sound (such as produced by pounding with a mallot on the door of the enclosure around the Dewar) produced large-n phase slips, we found the distribution of phase-slip sizes for the two flow directions to be relatively insensitive to smaller vibrations. For example, putting down the air springs of our vibration isolation system increased the vibrational input to the apparatus to a level which reduced the apparent phase-slip critical velocity by as much as 40%; yet it had no detectable effect on the average phase-slip size. Likewise, recooling the cell after warming through T_λ, or refilling the cell after emptying it of the helium sample at 4 K, had no effect on the distribution of sizes, or on the asymmetry with respect to the two flow directions. Thus we are led to believe that the quantum slip number n is basically determined by the details of aperture geometry, flow direction, and temperature.

There has been considerable discussion in the literature relating to experiments [1,11–13] which indicate that the critical velocity v_c for the onset of phase slips decreases linearly with temperature, falling to zero at an extrapolated temperature $T_0 \sim 2.5$ K. Such observations have been shown to be consistent with thermal activation of vortices over an energy barrier ΔE_b characterized by [1,11]

$$\Delta E_b = E_0(1 - v/v_{c0}) . \quad (2)$$

Figure 4 shows the temperature dependence of the average critical velocity at which phase slips occur for the two orifices reported in this paper. The data shown in this figure imply that the temperature dependence of v_c is not strictly linear [14] over the entire range and therefore the nucleation of phase slips must be more complex than the thermal activation given by Eq. (2). Furthermore, over the range where the measured v_c is linear ($T < 1$ K), the extrapolated value T_0 is found to be 1.9 ± 0.1 K for both orifices.

A stochastic analysis of the thermal activation process has shown [1] that T_0 is related to the activation energy E_0 in a simple way. Our value of T_0 differs significantly from that measured in other experiments [11–13,15]. Therefore we consider it likely that the precise value of T_0 is not intrinsic to helium, and the magnitude of the activation energy relevant to the nucleation problem is determined, in part, by the geometry or surface microstructure of the aperture. This is supported by studies of vortex nucleation around electron bubbles (of radius 1.6 nm) which indicate the presence of an energy barrier [16] of magnitude 3 K. This is smaller by a factor of at least 30 than the best estimate ($E_0/k_b = 106$ K) from orifice experiments [1,12].

A complication in the understanding of critical velocities arises from the observed existence above 0.9 K of a second, lower critical velocity branch for orifice 2 (see Fig. 4). This branch of v_c shows a weaker temperature dependence, and is characterized by multiple phase-slip events of size $n=1$ to 16. It is seen only for the flow

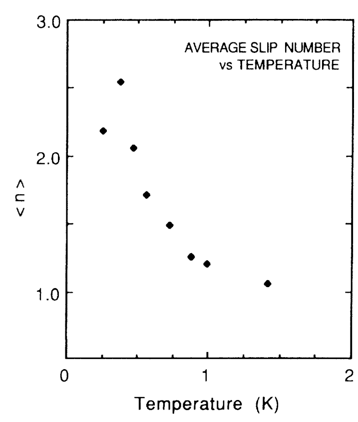

FIG. 3. The variation of the average slip number $\langle n \rangle$ with temperature for the direction exhibiting multiple slips in orifice 1. At a given temperature, the average is taken over all the events in that flow direction. Above 1.5 K it is not possible to resolve individual phase slips due to the low Q of the resonator. However, the critical velocity v_c can still be identified from a plot of oscillator amplitude vs applied drive.

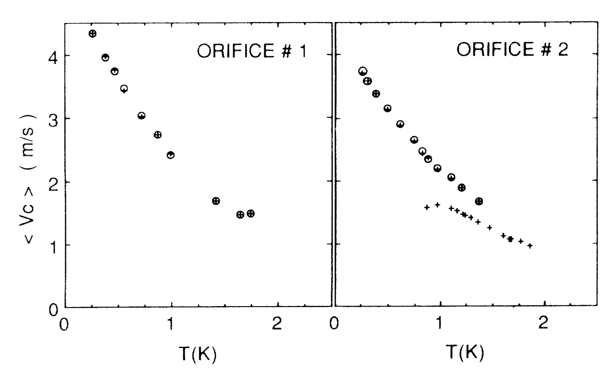

FIG. 4. Plots of phase-slip critical velocity vs temperature for the two orifices. Open circles and plusses correspond, respectively, to the negative and positive flow directions. The velocities shown are averages over the cross-sectional area of the orifice. Because of surface roughness, however, the local fluid velocity at the phase-slip nucleation site may be several times higher than the average.

direction which shows almost exclusively single 2π phase slips along the higher v_c branch. Once again, the average slip number $\langle n \rangle$ decreases with increasing temperature, being approximately 12 at 1 K and falling to 1 above 1.6 K. This second branch of v_c was not observed in the case of orifice 1, and it is not clear whether an easy interpretation of the two branches can be found in terms of two separate phase-slip nucleation centers. There is a considerable range of temperature over which the two branches coexist. In a given run, the system stays metastably in one of the two branches for periods lasting several hours, but can be made to switch to the other by applying to the resonator for a few seconds a drive 10^3 times higher than the minimum required to reach v_c. However, above 1.4 K, the higher v_c regime is completely preempted by the lower branch.

In conclusion, we have shown that quantized phase-slip events characterize the two orifices under study. The critical velocity v_c for the events is independent of flow direction. Below 1 K, thermal activation over a simple velocity-dependent energy barrier may adequately describe the temperature dependence of v_c. However, the values of the parameters which enter the theory are not identical to those determined in previous work, indicating that detailed surface structure or possibly the material of the surface determines the relevant energy barriers. At temperatures above 1 K, the thermal activation over the barrier given by Eq. (2) is an incomplete description of the nucleation process. The size of phase-slip events is found to depend on direction and geometry suggesting that multiple slip numbers occur due to complex reconnection processes. Elevated temperatures inhibit the multiple phase slips.

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[9] To derive Eq. (1), let a be the effective area of the diaphragm, k its stiffness in $\mathrm{N\,m^{-1}}$, A the amplitude of its displacement, and ΔA the loss in amplitude in a phase slip. Then $I_c = 2\pi f \rho_s a A_c$ and $\Delta E_{\rm obs} = \Delta(kA^2/2) = kA_c \Delta A$. Therefore $R = \Delta E_{\rm obs}/I_c = k\Delta A/2\pi f \rho_s a$. For an arbitrary displacement δX of the diaphragm, the change in the SQUID output δV is given by $\varepsilon \delta V/2ad^2 = k\,\delta X/a$. Thus, if $\Delta V_{\rm slip}$ is the change in SQUID output corresponding to ΔA, then $R = \varepsilon \Delta V_{\rm slip}/4\pi a \rho_s f d^2$.

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