A Chemical Potential "Battery" for Superfluid ^{4}He Weak Links

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Abstract. Research and development of superfluid weak links has been hindered by the absence of a source of dc chemical potential, similar to a simple battery or voltage source for analogous superconducting devices. We describe here a method for generating a dc chemical potential difference, $\Delta\mu$, across a weak link array in superfluid ^{4}He. The presence of a $\Delta\mu$ forces quantum oscillations at a Josephson frequency, selectable by the adjustment of input power to a heater. We discuss a case in which the frequency locks onto a resonance feature where it exhibits remarkable stability, and amplitude magnification by a factor of 40.

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Transient superfluid ^{4}He oscillations can be generated in a array of sub-micron apertures with the application of a pressure step, ΔP [1], or a temperature step ΔT [2]. The frequency of the oscillations f_J obeys the generalized Josephson frequency relation, $f_J = \Delta\mu/h$. Here $\Delta\mu = m_4(\Delta P/\rho - s\Delta T)$ is the chemical potential across the array, m_4 is the helium-4 atomic mass, ρ is the fluid density, and s its entropy per unit mass. Here we describe and demonstrate a method for producing continuous oscillations at a constant frequency, first proposed by Penanen and Chui [3]. A heater is used to maintain the constant $\Delta\mu$ which drives the oscillations. For reasons we do not yet fully understand, the system exhibits a high degree of stability at certain discrete frequencies. We propose a simple mathematical framework to describe the phenomenon.

Our experimental cell has been described in detail elsewhere [2]. A small volume of superfluid ^{4}He, itself immersed in a superfluid ^{4}He bath, is bounded on one side by the array (4225 apertures, nominally 70 nm in diameter, spaced on a 3 μm square lattice, in a 50 nm thick silicon nitride membrane), and on the other side by a flexible diaphragm. Inside the small volume there is a heater. The displacement of the diaphragm, detected using a SQUID-based transducer, indicates both the pressure ΔP across the array, and fluid flow through it.

If a small power W is applied to the heater, the inner volume warms, causing a net current to flow into it until steady state is reached. At this point $\Delta\mu = 0$ and $\Delta P = \rho s \Delta T$. Normal current I_n continues to flow, according to

$$I_n = -\frac{\rho_n\beta}{\eta}\left(\rho_n\frac{\Delta P}{\rho} + \rho_s s\Delta T\right). \qquad (1)$$

This reduces to $I_n = \rho_n\beta\Delta P/\eta$ when $\Delta\mu = 0$. Here ρ_n and ρ_s are the normal and superfluid densities, η is the normal fluid viscosity, and β is a geometrical factor. To maintain zero net current at steady state, a super current flows in the opposite direction: $I_s = -I_n$. The steady state ΔT is reached when the heater power is balanced by thermal conduction through the walls of the inner volume, heat carried out by normal flow, and heat required to "warm" incoming superflow:

$$W = \frac{\Delta T}{R} + sT\left(I_s - \frac{\rho_s}{\rho_n}I_n\right). \qquad (2)$$

Here R is the thermal resistance between the helium inside the inner volume at temperature $T + \Delta T$, and the helium outside, at constant temperature T.

If the heater power is slowly increased, ΔT, ΔP, I_n and I_s all increase according to the above relations, and $\Delta\mu = 0$ is maintained. I_s, however, cannot exceed the superfluid critical current I_c. Instead, when W exceeds a critical value, $\Delta\mu$ becomes non-zero and Josephson frequency oscillations, or "whistling," begins. A new steady state can be achieved in which the oscillations continue, $\Delta\mu = hf_J \neq 0$, the mean (dc) superflow $I_{s,\mathrm{dc}}$ replaces I_s in eq. 2, and $I_n = -I_{s,\mathrm{dc}}$.

Such a sequence of events is plotted in fig. 1. This data was taken at $T_\lambda - T = 1$ mK. For $t < 0$, a constant 63 nW is applied to the heater, and the measured pressure $\Delta P = 0.25$ Pa agrees with that predicted for when $\Delta\mu = 0$: $\Delta P = \rho sRW/\left(1 + \rho^2 s^2 TR\beta/\eta\right)$. Beginning at $t = 0$ the heater power is ramped linearly to a final 112 nW over 16 seconds. Initially ΔP rises to maintain $\Delta\mu = 0$ as the heater power rises, but at $t \simeq 1.5$ sec, the superfluid begins to oscillate. One might expect that transition to the whistling state to occur when I_s reaches I_c, at a fountain pressure $\Delta P_c = \eta I_c/\rho_n\beta_n$. An indepen-

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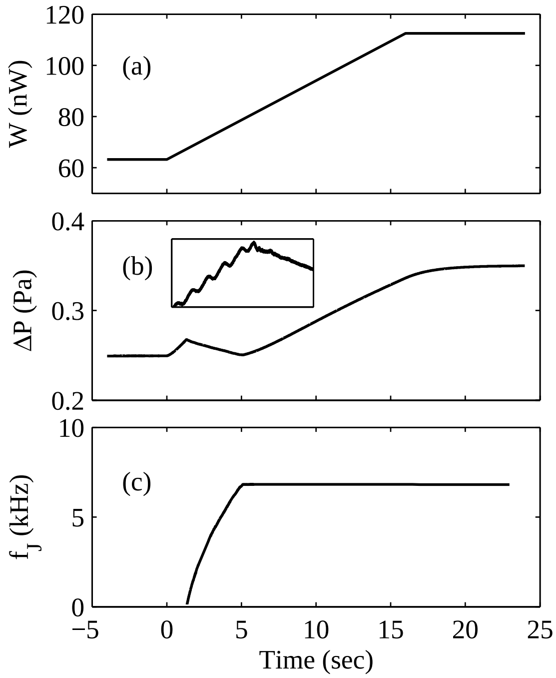

FIGURE 1. Method for producing a constant chemical potential difference and steady superfluid ^{4}He oscillations in a sub-micron aperture array. (a) Power delivered to the heater. (b) Evolution of the pressure across the array. The inset is a 0.4 sec wide section of data centered on the transition into the oscillating state that occurs at $t \simeq 1.5$ sec. (c) The oscillation frequency, shown locking into a stable frequency at $t \simeq 5$ sec.

dent measurement yielded $I_c = 5.6 \times 10^{-12}$ kg/sec. The corresponding ΔP_c is 2.0 Pa, almost a factor of 10 larger than the $\Delta P = 0.27$ Pa at which the system actually began to whistle. This suggests that the system is metastable at fountain pressures well below ΔP_c, and fluctuations can cause it to switch into a whistling state. Indeed, a close look at the transition at $t \simeq 1.5$ sec in the $\Delta P(t)$ plot of fig. 1b (see inset) reveals 23 Hz Helmholtz mode oscillations (not the same as Josephson frequency whistling) of increasing amplitude just before ΔP begins to drop and the superfluid begins to whistle.

As shown in fig. 1c, after the whistling begins, its frequency (and therefore $\Delta\mu$) rises steadily as the heater power continues to increase and ΔP drops. At $t \simeq 5$ sec, the frequency abruptly plateaus and ΔP begins rising again. For the next 11 seconds, ΔP continues to rise roughly linearly as the heater power rises, until $t = 16$ sec where the heater power reaches its final value and ΔP relaxes to a steady state value. From $t \simeq 5$ sec to $t = 16$ sec, the whistle, at 6.816 kHz with a width of only 0.25 Hz, drifts (downward) only 0.7 Hz, even while the heater power and pressure are increasing by 50%. We do not yet understand the reason for this remarkable stability. At $t > 24$ sec, the system remains in a steady state as long as the heater power is maintained. It seems likely that the 6.8 kHz value the whistle frequency locks onto is associated with a resonance feature of the system, but we have not yet determined the nature of this resonance. The amplitude of the 6.8 kHz signal is unexpectedly large – almost 40 times larger than would result from a current oscillation of amplitude I_c. From $t \simeq 5$ sec to $t = 16$ sec, the amplitude of the whistle varies (increases) by only 2%.

We have observed the system lock on to other frequencies as well. The frequency can be manipulated by varying the heater power – the steady state frequency achieved is dependent on the history of the heater power and on how fast it is changed.

The main unknown in the steady state formalism presented above is the mean superflow $I_{s,\mathrm{dc}}$. We believe this quantity will depend primarily on the whistle frequency f_J, and possibly its phase ϕ, with the spectrum $I_{s,\mathrm{dc}}(f_J, \phi)$ determined by the interaction of the superfluid oscillations with the resonant behavior of the overall system, perhaps similar in nature to the Fiske or Shapiro effects in superconductors [4], and in superfluid ^{3}He [5]. The intersection of this spectrum with the function $f_J(I_{s,\mathrm{dc}}, W)$ derived from the steady state formalism will determine an allowed set of Josephson frequencies with unstable, metastable, and stable branches which can be traversed by manipulation of the heater power W. If the nature of the $I_{s,\mathrm{dc}}(f_J, \phi)$ spectrum can be understood, it may be possible to design a cell to optimize stability and signal-to-noise.

We have demonstrated a method with which we have produced superfluid ^{4}He aperture array oscillations with a highly stable frequency and considerable amplitude magnification. The nature of the resonance behavior remains to be explained. This technique may prove to be an ideal method of operating a ^{4}He weak link device analogous to the dc-SQUID, which would be highly sensitive to rotation.

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