Shot Noise Acoustic Radiation Generated by Phase Slippage in Superfluid ^{4}He

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(Received 11 May 1998)

We present a model calculation and measurements of the acoustic radiation spectrum generated by thermally activated phase slippage driven by a constant pressure head. The calculation of the spectrum, which is similar to that for shot noise in an electrical current, takes into account the stochastic nature of phase-slip vortex nucleation. The observed acoustic amplitude depends on the driving pressure ΔP, the frequency ω, and the quality factor Q of a coupled cavity, as $(\Delta PQ/\omega^5)^{1/2}$, consistent with the model predictions. [S0031-9007(98)06925-7]

PACS numbers: 67.40.Mj, 67.40.Rp, 67.40.Vs

The simplest description of superfluidity focuses on flow without dissipation. However, it has long been recognized that dissipation-free superflow velocities are limited by some type of critical velocity [1]. In most situations, superfluid ^{4}He exhibits flow dissipation either through the continuous evolution of quantized vorticity [2] or through the simpler process of discrete phase slippage. The concept of discrete phase slippage was introduced by Anderson [3] to describe the case of superfluid passing through a small aperture. In the simplest scenario, the superflow nucleates a quantized vortex line which passes across the flow path. The growth of the vortex removes energy from the ambient flow and lowers the average superfluid velocity by $\delta v_s = \kappa/l$. Here, $\kappa = h/m_4$ is the quantum of circulation and l is an effective length of the aperture. This process decreases the quantum phase difference across the aperture by 2π. In recent years [4,5], single 2π phase slips have been detected when a small aperture is used as the inductive element in a low frequency hydrodynamic oscillator. The phase-slip events remove discrete amounts of energy from the oscillator on a time scale short compared to the period.

The pressure dependence of the limiting velocity in dc flow has also been interpreted [6] in terms of discrete phase-slip processes, although the presence of phase slips in dc flow has not been directly detected. If only 2π phase slips occur when a pressure head ΔP forces superfluid through an aperture, the average number of phase slips per unit time is given by the Josephson frequency relation [3]: $f_j = \Delta P/\rho\kappa$, where ρ is the total liquid density [7].

This model implies that when discrete phase slippage occurs in pressure-head driven flow, the time resolved velocity resembles an asymmetric sawtooth. In one cycle the velocity increases linearly in time due to the pressure-head acceleration until, at some critical velocity, the flow velocity abruptly decreases by δv_s [8]. The fluctuating velocity field in the aperture should radiate sound into the surrounding fluid. In order to make a more direct test of this model for dc dissipation, we have designed an experiment to detect and characterize acoustic radiation resulting from the discrete phase-slip process.

Since the vortex nucleation process is stochastic, the critical velocity for a given phase slip is not fixed but can be characterized by a distribution function of width Δv_c centered around the average critical velocity [4] $\langle v_c\rangle$. We are concerned here with apertures characterized by $\Delta v_c > \delta v_s$. In this case, the phase slips occur at the average rate f_j, but are uncorrelated in time. The expected acoustic spectrum is then analogous to shot noise in a dc electrical current flowing through a wire [9]. The discrete size of the electron charge plays the role of the discrete size of the velocity decrement δv_s. The dc current, which determines the average rate of electrons passing a point in the wire, plays the role of the average frequency f_j. The discrete size of the electron and the randomness of its arrival time lead to variations in the current having a broad or "white" spectrum. Analogously, the expected acoustic field resulting from quantized but uncorrelated phase slips should be similarly broad [10].

To predict the acoustic spectrum, we consider a small aperture placed at one end of a cylindrical cavity similar to an organ pipe, closed at one end and open at the other. The aperture is positioned at the center of the closed end. Assume that superfluid is accelerated through the aperture with a constant pressure head (i.e., Josephson frequency) applied across it. Each phase slip, which causes a nearly instantaneous decrease in the superfluid velocity at the aperture, is equivalent to a δ-function deceleration. The combination of these two accelerations pumps energy into the acoustic pressure field within the resonator. We consider a single resonant mode of the cavity and describe the mode as a damped simple harmonic oscillator. The equation of motion for the acoustic pressure p at the end of the resonator containing the aperture is given by

$$\frac{d^2p}{dt^2} + \frac{\omega_i}{Q_i}\frac{dp}{dt} + \omega_i^2 p = \frac{2\rho c^2 a}{V_r}\,\delta v_s\left[f_J - \sum_n \delta(t-\tau_n)\right], \quad (1)$$

where ω_i is the resonant frequency of the ith normal mode, Q_i is the quality factor of the resonance, a is the area of the aperture, V_r is the volume of the resonator, and c is the speed of first sound. The first and second terms on the right-hand side represent accelerations due to the applied pressure-head and instantaneous phase slips, respectively. The τ_n's, which are the times phase slips occur, have an average spacing of f_j^{-1} and are taken to be uncorrelated. The δ functions on the right-hand side are then analogous to a shot-noise process [9]. By taking a Fourier transform, Eq. (1) is solved for the response function $p(\omega)$. The rms pressure, p_{rms}, is calculated by integrating $|p(\omega)|^2$ over all frequencies and taking a square root. Using the relationship between pressure and displacement in an acoustic standing wave, the rms displacement at the free end of the resonator is then found to be

$$x_{\mathrm{rms}} = \frac{p_{\mathrm{rms}}}{\rho c \omega_i} = \frac{ac}{V_r}\,\delta v_s \sqrt{\frac{\pi Q_i \Delta P}{\rho \kappa \omega_i^5}}\,. \tag{2}$$

Our experiment consists of detecting the rms acoustic displacement at the free end of the resonator and testing to see if it scales as $(\Delta P Q_i/\omega_i^5)^{1/2}$.

A schematic drawing of our apparatus, which is described in detail elsewhere [11], is shown in Fig. 1. The experiment involves driving superflow through a submicrometer sized aperture *at fixed pressure head* and, therefore, a fixed average rate of phase slippage. The aperture is placed at one end of an acoustic cavity. Acoustic radiation from phase slippage in the aperture pumps energy into the resonant modes of the cavity. By monitoring the acoustic displacements at one of the cavity resonances f_i, we are able to check the predictions of Eq. (2).

The acoustic detection section [11] consists of a $\frac{1}{4}$-wave acoustic resonator with the aperture mounted at one end and a capacitance microphone at the other. In the frequency range of interest, 1–3 kHz, the microphone membrane behaves nearly as a free end. This is the result of two facts: (1) The space above the microphone membrane is evacuated and (2) the low mass and spring constant of the membrane. By biasing the microphone at constant charge through a large resistor [12], the acoustic displacement is turned into an oscillating voltage which is amplified and passed through a narrow band filter ($\Delta f \sim 4$ Hz) centered on the cavity resonance. The rms value of this voltage is a measure of x_{rms} in Eq. (2).

The pressure regulation/flow driving section of the cell [11,13] consists of two flexible membranes coupled by a feedback system. A pressure head across the aperture causes a displacement of the pressure gauge membrane, which is detected by a capacitance bridge. (The pressure sensitivity, $8 \mu Pa_{\mathrm{rms}}/\mathrm{Hz}^{1/2}$, is preamp noise dominated.) The flow driver is driven by a voltage which is the (integrated and amplified) difference between the bridge and a reference signal. In this configuration, the feedback maintains the pressure gauge at fixed displacement (pressure head) while driving flow through the aperture. By sweeping the pressure-head reference voltage, the acoustic displacement in a particular resonator mode is measured as a function of pressure head.

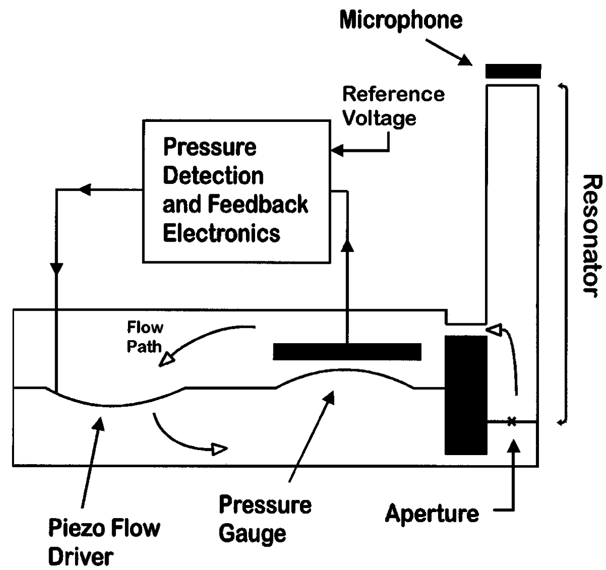

FIG. 1. Schematic of the experimental cell. The microphone consists of a metallized 24 μm thick Kapton membrane held approximately 25 μm from a rigid brass electrode. The membrane is circular and 0.95 cm in diameter. The space between the membrane and brass electrode is evacuated. The aperture is made by etching through a 50 nm thick Si-N membrane which is grown on one side of a Si chip [18]. The pressure gauge is a metallized 8 μm thick Kapton membrane held approximately 18 μm from a rigid copper plate. The membrane is circular and 4.4 cm in diameter. The flow driver is a circular brass foil, 250 μm thick and 3 cm in diameter, which is actuated by a piezoelectric element. Drive voltages applied to the piezo cause the brass foil to bow the change the relative volume of the two sides of the cell to an amount proportional to the applied voltage. This action is used to both generate a pressure head and drive flow through the aperture. The drive voltage is provided by the feedback electronics.

In order to isolate the resonant cavity from external sources of acoustic drive, the Dewar, which contains a single-shot ^{3}He refrigerator, is hung from the ceiling by rubber cords. Also, the experimental cell is itself hung by springs from the ^{3}He pot. This isolation lowers the external drive so that the microphone sensitivity is limited by preamplifier noise.

In addition to looking for the phase slip acoustic radiation, the feedback system can also be used to measure the average critical velocity as a function of pressure head [11,13]. We fix the reference pressure and determine the associated critical current from the rate of motion of the driver. From the function $v_c(\Delta P)$, we can obtain Δv_c, the width of the critical velocity distribution function, from the formula [6]

$$\Delta v_c = \frac{2}{\ln 2}\,\frac{d v_c}{d(\ln \Delta P)}\,. \tag{3}$$

We have studied the acoustic radiation from three apertures. The first is a 0.42 μm $\times$ 0.12 μm aperture. The average critical velocity, for this aperture, scales as $v_c \sim (1 - T/2.2\ \mathrm{K})$, which is the typical signature of the 2π phase slip process [4]. Assuming as effective length [14] of 215 nm and using Eq. (3), we find to within the experimental scatter, $\Delta v_c/\delta v_s$ is nearly temperature independent with an average value of 1.4. Thus the phase slips for this aperture are uncorrelated in time, and Eq. (2) applies.

Figure 2(a) displays the rms acoustic amplitude as a function of pressure-head at several temperatures. The scatter in the acoustic signal, which is large compared to electronic noise, is generated by the stochastic nature of the phase slips. Although v_c and $\Delta v_c/\delta v_s$ do not vary appreciably over this temperature range, the magnitude of the acoustic signal varies by nearly a factor of 2 at fixed pressure head ΔP. Also, a function of ΔP, the three curves seem to follow the same functional form. These observations can be understood from Eq. (2). Figure 2(b) shows the acoustic amplitudes scaled by $Q^{1/2}$ (the Q's were measured from the free ringing decay time of the resonance). Now, all the data falls on a universal curve in excellent agreement with the $Q^{1/2}$ dependence predicted by Eq. (2). For comparison, a smooth curve given by $\Delta P^{1/2}$ is drawn through the data showing that it scales as predicted by Eq. (2).

Similar measurements were carried out on a second aperture (transverse dimensions of 0.18 μm $\times$ 4.2 μm).

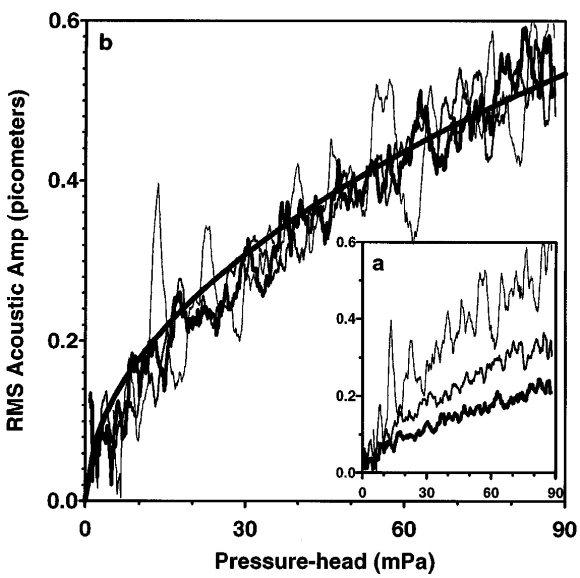

FIG. 2. [(a), inset] Acoustic displacement versus ΔP for the 0.42 μm $\times$ 0.12 μm aperture at 0.25, 0.35, and 0.45 K (top to bottom). The electronic noise, approximately $0.01 pm_{\rm rms}$, is small compared to the intrinsic fluctuations in the signal. (b) Acoustic displacement at the same temperatures after rescaling the 0.35 and 0.45 K data by $(Q_{0.25}/Q_{0.35})^{1/2}$ and $(Q_{0.25}/Q_{0.45})^{1/2}$, respectively. The measured Q's are given by $Q_{0.25} = 13\,000$, $Q_{0.35} = 5200$, and $Q_{0.45} = 2200$. Plotting the data in this way shows the excellent agreement with the $Q^{1/2}$ dependence predicted by Eq. (2). After scaling, the three sets of data fall on a universal curve. The smooth curve, which is $\sim\Delta P^{1/2}$, shows that the data is in agreement with the $\Delta P^{1/2}$ dependence of Eq. (2).

Figure 3 shows the measured acoustic signal as a function of pressure head at 0.30 and 0.40 K. In this temperature range the second aperture displayed a nearly temperature independent critical velocity ($\langle v_c\rangle \sim 3.2$ m/s), a characteristic of apertures that generates multiple quantization of the phase-slip amplitude [15]. It can be shown [9] that the presence of multiple phase slippage in the theory does not change the result given in Eq. (2). The dashed lines in Fig. 3, which are $\sim\Delta P^{1/2}$, show that the data scale as predicted in Eq. (2). For this aperture the acoustic amplitude also scaled as $Q^{1/2}$, within a few percent scatter of the measurement.

A third aperture, (0.13 μm $\times$ 5.3 μm), was used to test the $\omega_i^{-5/2}$ scaling predicted in Eq. (2) by using two resonant modes of the cavity. Figure 4 shows the rms acoustic displacement generated by this aperture in both the first and second plane wave modes of the cavity (1070 and 2900 Hz, respectively). The solid line, which is $\sim\Delta P^{1/2}$, shows good agreement with the scaling predicted by Eq. (2). The data from the second mode are replotted, multiplied by $(Q_2/Q_1)^{1/2}(\omega_1/\omega_2)^{5/2} = 10.2$. Nearly all of this factor comes from the ω dependence. The rescaled points fall nearly on the same curve as the lowest mode, in agreement with the $\omega^{-5/2}$ prediction of Eq. (2).

In conclusion, we have detected acoustic radiation generated by phase slippage in a submicron aperture.

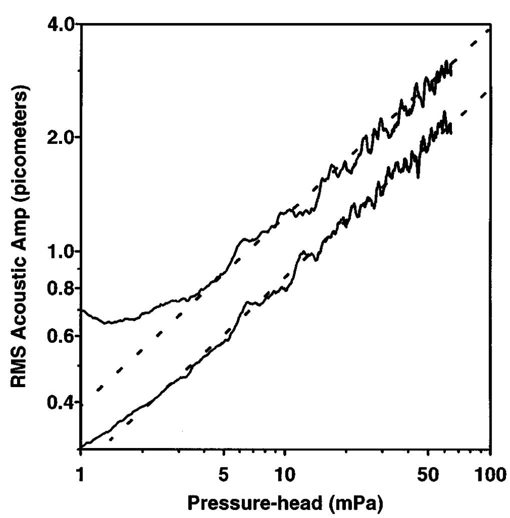

FIG. 3. Acoustic displacement versus ΔP for the 0.18 μm $\times$ 4.2 μm aperture at 0.30 and 0.40 K (top to bottom). The dashed lines through the data are $\sim\Delta P^{1/2}$, showing excellent agreement with the $\Delta P^{1/2}$ dependence depicted in Eq. (2). By fitting lines to $\log(x_{\rm rms})$ vs $\log(\Delta P)$, the acoustic amplitudes are found to be $x_{\rm rms} = (0.397 \pm 0.005)[\Delta P({\rm mPa})]^{(0.493\pm0.003)}$ and $(0.253 \pm 0.005)[\Delta P({\rm mPa})]^{(0.522\pm0.003)} pm_{\rm rms}$, respectively. Both of the fitted exponents are in close agreement with the value of 0.50 predicted by Eq. (4). Also, the ratio of the two amplitudes is 1.57 ± 0.05, which is in excellent agreement with the ratio predicted by the measured Q's of $(8200/3300)^{1/2} = 1.58$. The critical velocity in this aperture was measured to be 3.2 m/s and to be roughly temperature independent.

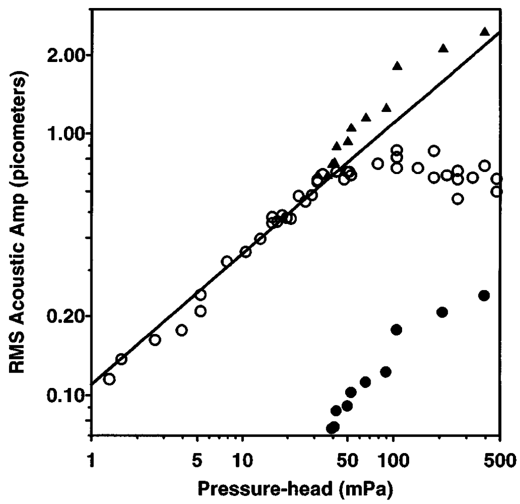

FIG. 4. Acoustic displacement versus ΔP for the 0.13 μm $\times$ 5.3 μm aperture at 0.30 K. The open circles and closed circles are the acoustic displacement in the first (1070 Hz) and second (2900 Hz) plane wave modes, respectively. The solid line is $\sim\Delta P^{1/2}$. Comparison with the data again confirms the $\Delta P^{1/2}$ scaling of Eq. (2). [A fit to the 1070 Hz data below 100 mPa yields $x_{\rm rms} = 0.104[P(\rm mPa)]^{0.505} pm_{\rm rms}$. The data at higher pressure heads falls significantly below the fitted line, but the falloff occurs at a pressure head where the average time between slips is comparable to the time required for the vortex ring to pass halfway across the aperture [19]. In this case, the sawtooth velocity field in the aperture should decrease in amplitude, thus decreasing the acoustic radiation amplitude.] The closed triangles are the 2900 Hz data scaled according to Eq. (2) (see the text for description of the scaling). The agreement between the fitted line and the scaled 2900 Hz data provide evidence that the $\omega^{-5/2}$ prediction of Eq. (2) is correct.

The acoustic amplitude within a resonant cavity is found to scale with the control variables (ΔP, Q_i, and ω_i), consistent with [16] Eq. (2). These observations confirm our assertion that the stochastic nature of the phase slippage produces a white noise background reminiscent of shot noise in an electrical current. This provides further evidence that the dissipation in pressure-head-driven superflow is due to phase slippage.

All three of the apertures explored in this paper have $\Delta v_c > \delta v_s$ and exhibit broad spectra due to the lack of correlation between the phase slips. However, if $\Delta v_c < \delta v_s$, the acoustic spectrum in the cavity should begin to exhibit sharp peaks at multiples of f_j. If such an aperture is used in the type of apparatus described here, the Josephson frequency relation for ^{4}He could then be directly verified [17].

A. Loshak and N. Bruckner supplied the apertures for this experiment. We have benefited from discussions with K. Schwab, S. Vitale, and J. C. Davis. This experiment was partially supported by the NSF, the ONR, and NASA.

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