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Evidence for Thermally Activated Dissipation in Flowing Superfluid ^{3}He

J. P. Pekola, J. C. Davis, and R. E. Packard

Department of Physics, University of California, Berkeley, California 94720

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We report on observations of the onset of dissipation in superfluid ^{3}He flowing through a single cylindrical channel of diameter $d = 0.7\,\mu m$ *and length* $L = 6.0\,\mu m$. *We propose a model in which thermally activated phase slips in the order parameter can account for the observed onset of dissipation.*

1. INTRODUCTION

The mechanism for the onset of dissipation in quantum fluids (i.e., superfluids and superconductors) is a problem of fundamental interest. It is believed that thermal fluctuations are responsible for the onset of flow dissipation in both superfluid ^{4}He (He II)[1,2] and in superconducting microbridges.[3-8] In both cases, near T_c, the dissipation is caused by thermal activation of phase slips in the order parameter of the quantum fluid.

This paper reports on measurements of the onset of flow dissipation in superfluid ^{3}He flowing in a cylindrical channel of diameter $d = 0.7\,\mu m$ and length $L = 6.0\,\mu m$. This is the first investigation to study such effects using a *single* well-characterized channel.[9] The small diameter prevents quantized vortex creation from masking pair-breaking phenomena. Also, in restricted volumes of ^{3}He such as this one, thermal fluctuation effects are predicted to play a significant role.[10] This system is very similar to a superconducting microbridge.

Several experiments to investigate the flow of superfluid ^{3}He have shown dissipation, which seems not to be consistent with the simplest pair-breaking mechanism. These include measurements of both ac and dc flows through a rectangular cross-section superleak,[11,12] of ac flow through multiple micropores at very high-pressure resolution,[13] and, more recently, measurements of flow in thin superfluid films.[14,15]

2. EXPERIMENT AND DATA

The microchannel used in this experiment is formed by etching away the radiation-damaged track produced by the passage of a single ^{252}Cf fission fragment through a 6-μm polycarbonate film.[16] This film, containing the channel, is glued to a brass ring and is mounted in the wall separating two chambers containing ^3He as shown in Fig. 1. Also contained in this wall is a 10-μm-thick, flexible diaphragm, which is metallized on both sides. Fixed metal plates positioned 20 μm away are used to apply electrostatic forces to this diaphragm and also to measure the resulting motion by capacitive methods. The capacitance bridge can detect average motion in the diaphragm of 10^{-12} m, which translates to a pressure sensitivity of 10^{-4} Pa.

This cell is cooled by a conventional nuclear demagnetization cryostat. Thermometry is performed using pulsed NMR on ^{195}Pt powder. The cell is filled through two capillary tubes, which join at a small reservoir mounted on the mixing chamber of a dilution refrigerator. Because of the large viscosity of the normal ^3He in these tubes (which are about 3 m long in total and have an inner diameter of 100 μm), the current through this flow path (which is in parallel to the microchannel) is negligible in comparison

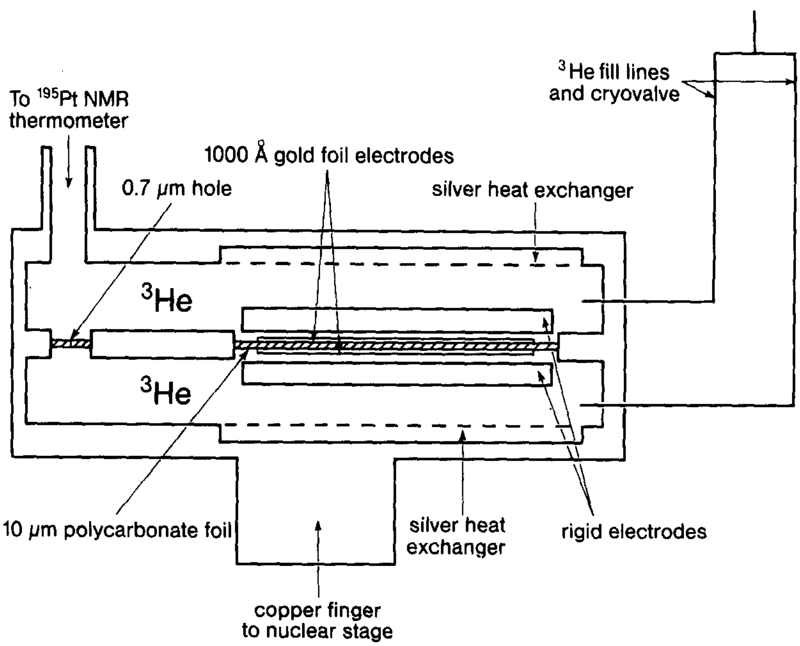

Fig. 1. The experimental cell used in this study.

to the supercurrents measured. This means that any pressure difference between the two chambers relaxes by superflow through the microchannel.

The principle of the measurement technique is as follows. When the diaphragm is at rest in equilibrium and when $T < T_c$, a voltage is applied between one of the fixed electrodes and one side of the diaphragm. Under the influence of the resultant electrostatic force the diaphragm moves at a rate $\dot{X}$ toward its new equilibrium position at X_0, which is the position where the electrostatic force is balanced by the elastic restoring force of the flexible diaphragm. The supercurrent density J_s is related to $\dot{X}$ through mass conservation

$$J_s = \rho(A/a)\dot{X} \qquad (1)$$

where A is the effective diaphragm area, a is the cross-sectional area of the microchannel, and ρ is the density of the fluid. The pressure difference ΔP between chambers is directly related to the distance of the diaphragm from its equilibrium position $X - X_0$ through the measured spring constant λ of the diaphragm

$$\Delta P = -\lambda(X - X_0) \qquad (2)$$

A measurement of this form yields the diaphragm position X as a function of time t. The raw data are smoothed by a fifth-order polynomial fit to $X(t)$ and differentiation with respect to time yields the function $\dot{X}(t)$. Thus, from Eqs. (1) and (2), and knowing X_0, the supercurrent density J_s as a function of the pressure difference ΔP can be found, yielding the function $J_s(\Delta P)$, which is of central interest.

Shown in Fig. 2 are examples of $J_s(\Delta P)$ curves measured using the above method at three different temperatures. The significant features of these cures are:

1. At pressures $\Delta P \geq 5$ mPa the supercurrent is independent of pressure. We identify this saturation of J_s as the Ginzburg-Landau (GL) depairing critical current J_s^c. The magnitude and temperature dependence of this saturation current agree well with GL model calculations for a polar phase of superfluid ^{3}He in a microchannel of this size with diffuse scattering boundary conditions.[17]

2. At pressures below $\Delta P = 5$ mPa the supercurrent J_s falls as a function of ΔP to reach a value $J_s(0)$ at the point of minimum pressure resolution when $\Delta P \sim 0.1$ mPa. In this experiment, $J_s(0)$ is the current density at the onset of observable dissipation.

3. The ratio of $J_s(0)$ to the saturation current density J_s^c at a given temperature falls as temperature increases. This enhanced suppression of the current density with increasing temperatures (after the temperature

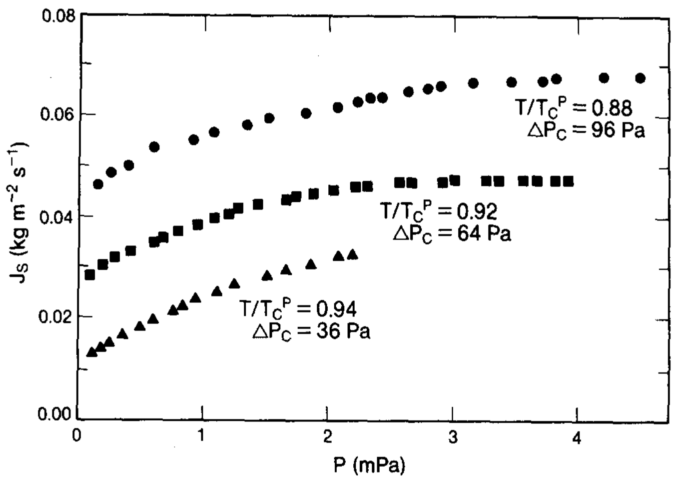

Fig. 2. The supercurrent density J_s as a function of the pressure difference ΔP between the two chambers containing ^{3}He. The $J_s(\Delta P)$ is shown at three different temperature below the transition temperature T_c^p inside the microchannel.

dependence of the superfluid density has been factored out) suggests that the dissipation process is thermally activated.

4. The pressures at which these effects are observed are at least three orders of magnitude below the pressure ΔP_c that would be measured if normal currents of the same magnitude were flowing through the microchannel.* The minimum length of a normal region inside the microchannel must be of the order of the coherence length $\xi(T)$ and the minimum ΔP that would be measured under conditions of a *static* normal region is $\Delta P = \Delta P_c(\xi(T)/L)$, which is of the order of 1 Pa. Since the measured ΔP values are still far below this, an analysis of the data should consider a dynamical process.

3. ANALYSIS

Since in ^{4}He and in superconductors the mechanism for the onset of flow dissipation is thought to be thermal activation of phase slips, it is natural to ask if any similar analysis will explain the observed onset of dissipation in superfluid ^{3}He. First it is necessary to establish the nature of the superfluid state in the microchannel. We model this system in the

*The ΔP_c is calculated using Rice's formula for collisionless Fermi liquid flow,[18] $\Delta P_c = J_s^c p_F L/m_3 d$. Here m_3 is the mass of the ^{3}He atom and p_F is the Fermi momentum. This gives values for ΔP_c in the range 30–100 Pa.

Ginzburg-Landau formalism, taking a simple diagonal order parameter of the form

$$\mathbf{A}=\begin{bmatrix}\Delta_\perp & & \\ & \Delta_\perp & \\ & & \Delta_\parallel\end{bmatrix}e^{iqz} \tag{3}$$

Here $\Delta_\perp$ is the energy gap of the superfluid perpendicular to the flow axis and $\Delta_\parallel$ is the gap parallel to it. If one assumes diffuse scattering boundary conditions, it is found upon numerical calculation[17] of the free energy involved that the state in the microchannel at $T/T_c > 0.6$ and at zero pressure is the polar state, i.e., the only term in A not equal to zero is $\Delta_\parallel e^{iqz}$. The state is also predicted to be the polar state close to T_c by Barton and Moore[19] and Jacobsen and Smith.[20]

A phase slip may occur in a region where a local fluctuation permits the otherwise smoothly varying quantum phase abruptly to change its value by 2π, when the order parameter collapses to zero along a length of the microchannel of order $\xi(T)=\xi_0/(1-T/T_c)^{1/2}$. For the case of diffuse boundary scattering, the order parameter falls to zero at the walls in a length again of order $\xi(T)$. This implies that the effective diameter over which the superfluid has its bulk value is $d' \sim d-2\xi(T)$. Our experiments are carried out at zero pressure, where $\xi_0 \sim 750$ Å. Thus, $d'=\xi(T)$ at a temperature $T/T_c=0.9$, since $d=0.7\ \mu$m. In this temperature region thermal fluctuations, sufficient to cause the order parameter to collapse in a volume of order $\xi(T)^3$, can result in the complete absence of superfluid from a region of the microchannel of length $\xi(T)$ and diameter d, thus causing dissipation due to the normal current flowing through this region.

The thermal fluctuations determine the frequency f of these phase slip events. Following Little[3] (superconductors) and Langer and Fisher[1] (He II), we assume that f is proportional to the Boltzmann factor, i.e.,

$$f=f_0\exp(-\Delta E/k_D T) \tag{4}$$

Here ΔE is the energy barrier that must be surpassed if the fluid in a volume $V=a\xi(T)$ is to go from the superfluid to the normal state. This barrier separates two macroscopic flow states of phase difference 2π, and f_0 is an intrinsic attempt frequency. We use the Legendre transformed free energy functional G in the Ginzburg-Landau theory: $G=G(\rho_s, J_s)$, where ρ_s is the superfluid density and J_s is the supercurrent density (see Fig. 3). To find G, we use a trial polar phase order parameter, whose value falls to zero at the walls over length $\xi(T)$, in a variational calculation. The stable superfluid state at the local minimum of G is separated from the normal state by a barrier ΔG, which is the difference between the maximum and the local minimum of G. The value of ΔG depends on J_s and on T as

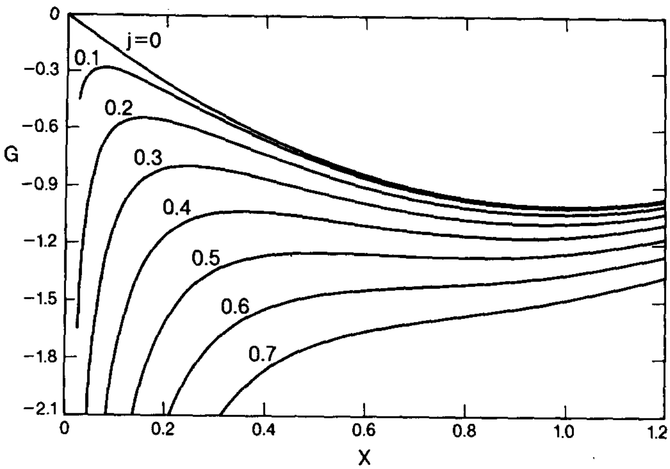

Fig. 3. The dimensionless free energy G at different current densities j as a function of the dimensionless superfluid density x. The critical current j_c, at which the minimum of G vanishes, is $j_c = (2/3)^{3/2} \sim 0.54$. Here $G = -x(2-x) - j^2/x$. The dimensionally correct free energy is obtained by multiplying G by $3\alpha^2/20\beta$, where α and β are the well-known Ginzburg-Landau parameters: $\alpha = \frac{1}{3}N(0)(1 - T/T_c)$ and $\beta = 7\zeta(3)/240\pi^2]N(0)/k_B T_c$. See text for further definitions.

$(1 - J_s/J_s^c)^{3/2}(1 - T/T_c^p)^2$ near T_c^p, where T_c^p and J_s^c are the critical temperature and GL critical current density inside the channel, both of which are reduced from their bulk values by restricted geometry effects.[17,20] The energy difference between these two states in a volume $\xi(T)a$ is $\Delta E = \Delta G(\xi(T)a)$. This includes only the difference in the condensation energy; however, taking the bending energy into account changes ΔE only slightly. We have found from numerical calculations using the polar phase order parameter that

$$\Delta E/k_B T \sim R[4\pi^2 N(0) k_B T_c/7\zeta(3)] \times (a\xi_0)(1 - T/T_c^p)^{3/2}(1 - J_s/J_s^c)^{3/2} \quad (5)$$

In this expression $N(0)$ is the density of states for one spin direction at the Fermi surface, k_B is the Boltzmann constant, and ζ is the Riemann zeta function. The factor R accounts for the reduction in the condensation energy density inside the channel compared to its bulk value. This reduction results because (1) the z component of the order parameter is the only nonzero component (i.e., the polar phase), and (2) the order parameter falls to zero at the walls over a distance of order $\xi(T)$ which is of the same magnitude as the channel radius. The value of R is found from numerical calculations to be $R = 0.14$.

Equation (5) shows that the energy barrier for phase slip events is lowest in narrow channels and close to the critical temperature and critical current.

Besides J_s, the other experimental observable is ΔP, the pressure drop across the microchannel. In the absence of thermal gradients and magnetic fields one can express the potential difference $\Delta\mu$ across the microchannel as $2m_3\Delta P/\rho$. If the phase of the order parameter undergoes 2π slips at a frequency f', then $\Delta\mu$ is given by the relation[21]

$$\Delta\mu = 2\pi f'\hbar = (2m_3/\rho)\,\Delta P. \tag{6}$$

Setting f' from Eq. (6) equal to the frequency f in Eq. (4), one finds that the fluctuation-induced ΔP is

$$\Delta P = (\pi\hbar\rho/m_3)f_0 \exp(-\Delta E/k_B T) \tag{7}$$

Combining Eqs. (5) and (7) gives

$$1-\frac{J_s}{J_s^c}=\left(\frac{1-T}{T_c^p}\right)^{-1}\left[\frac{7\zeta(3)\ln(\pi\hbar\rho f_0/m_3\Delta P)}{4\pi^2 N(0)k_B T_c Ra\xi_0}\right]^{2/3} \tag{8}$$

This allows us to compare how much the supercurrent density is reduced from its critical value due to thermal fluctuations at the minimum pressure resolvable, i.e., at the onset of observed dissipation.

In Fig. 4 we show the measured temperature dependence of $1-J_s(0)/J_s^c$. Also shown is a plot of this function calculated from Eq. (8) using $\Delta P = 10^{-4}$ Pa and $f_0 = \nu_0 La$, where $\nu_0 = n_3/\tau$.[4] Here n_3 is the number density of ^{3}He atoms and τ is the inelastic scattering time of quasiparticles. The calculated function $1-J_s(0)/J_s^c$ is insensitive to changes of many orders of magnitude in f_0. A second curve shows the results using a bulk B-phase ^{3}He order parameter in the calculation. Note that since it is the ratio $J_s(0)/J_s^c$ that is calculated and measured, the normal $(1-T/T_c)^{3/2}$ dependence of the critical current has been removed from these plots and the remaining temperature dependence is ascribed to thermal activation.

The reasonable agreement between the polar phase calculation and the data suggests that a thermal activation process may indeed explain the observed onset of dissipation. However, the model does not reproduce the entire observed $J_s(\Delta P)$ curve currectly, possibly because it does not take into consideration the interaction of phase slip regions in the microchannel.

To summarize, we have measured the current at which detectable dissipation in the dc flow of superfluid ^{3}He in a very narrow channel first appears. The magnitude and the temperature dependence of this current $J_s(0)$ suggest that the dissipation is thermally activated in the vicinity of the reduced superfluid transition temperature T_c^p.

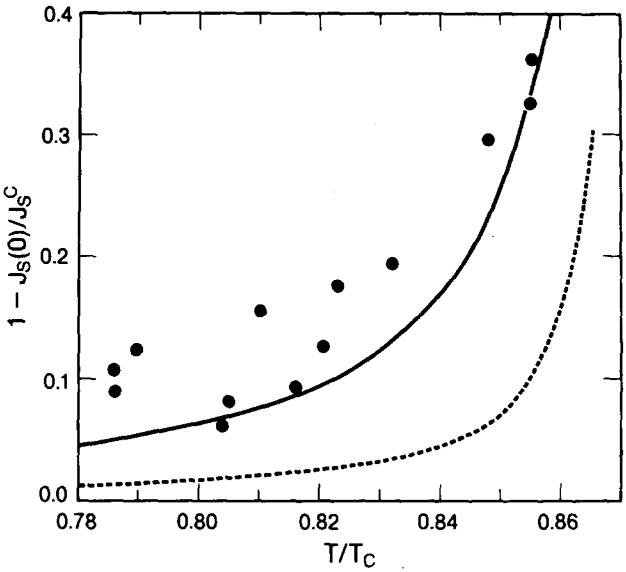

Fig. 4. The temperature dependence of the function $1 - J_s(0)/J_s^c$: (●) as measured (solid circles), (- -) as calculated from the bulk B-phase order parameter setting $T_c^{\text{bulk}} = T_c^p$, and (—) as calculated using the polar phase order parameter in the restricted geometry, where the $(1 - T/T_c^p)$ factor appears naturally from the calculation. Note that the measured effect turns on in a temperature region well below T_c^{bulk}, indicating that the dissipation is due to a process *inside* the microchannel.

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REFERENCES

1. J. S. Langer and M. E. Fisher, *Phys. Rev. Lett.* **19**, 560 (1967).
2. J. S. Langer and J. D. Reppy, in: *Progress in Low Temperature Physics*, Vol. VI, C. J. Gorter, ed. (North-Holland, 1970), p. 34.
3. W. A. Little, *Phys. Rev.* **156**, 396 (1967).
4. J. S. Langer and V. Ambegaokar, *Phys. Rev.* **164**, 498 (1967).
5. D. E. McCumber and B. I. Halperin, *Phys. Rev. B* **1**, 1054 (1970).
6. W. W. Webb and R. J. Warburton, *Phys. Rev. Lett.* **205**, 461 (1968).
7. J. E. Lukens, R. J. Warburton, and W. W. Webb, *Phys. Rev. Lett.* **25**, 1180 (1970).
8. P. E. Lindelof, *Rep. Prog. Phys.* **44**, 949 (1981).

9. M. T. Manninen and J. P. Pekola, *J. Low Temp. Phys.* **52**, 497 (1983).
10. K. B. Efetov and M. M. Salomaa, *J. Low Temp. Phys.* **42**, 35 (1981).
11. A. J. Dahm, D. S. Betts, D. F. Grewer, J. Hutchins, J. Saunders, and W. S. Truscott, *Phys. Rev. Lett.* **45**, 1411 (1980).
12. Ren-Zhi-Ling, D. S. Betts, and D. F. Brewer, *Phys. Rev. Lett.* **53**, 930 (1984).
13. V. Kotsubo, K. D. Hahn, and J. M. Parpia, *Phys. Rev. Lett.* **58**, 804 (1987).
14. J. G. Daunt, R. F. Harris-Lowe, J. P. Harrison, A. Sacharjda, T. Seeto, S. C. Steel, R. R. Turkington, and P. Zawadzki, *Jpn. J. Appl. Phys.* **26**, 145 (1987).
15. J. C. Davis, A. Amar, J. P. Pekola, and R. E. Packard, *Jpn. J. Appl. Phys.* **26**, 147 (1987).
16. R. E. Packard, J. P. Pekola, P. B. Price, R. N. R. Spohr, K. H. Westmacott, and Zhu Yu-Qun, *Rev. Sci. Instrum.* **57**, 1654 (1986).
17. J. P. Pekola, J. C. Davis, Zhu Yu-Qun, R. N. R. Spohr, P. B. Price, and R. E. Packard, *J. Low Temp. Phys.* **67**, 47 (1987).
18. M. J. Rice, *Phys. Rev.* **165**, 288 (1968).
19. G. Barton and M. A. Moore, *J. Low Temp. Phys.* **21**, 489 (1975).
20. K. W. Jacobsen and H. Smith, *J. Low Temp. Phys.* **67**, 83 (1987).
21. P. W. Anderson, *Rev. Mod. Phys.* **38**, 298 (1966).