AN UPPER LIMIT TO ROTATIONAL COOLING IN SUPERFLUID HELIUM*

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We present the results of a calculation and experiment which show that rotation of He II in packed powder will not produce a decrease in temperature greater than 2×10^{-5} °K. This is in contrast to a recent prediction.

A recent calculation predicted [1] that the adiabatic rotation of superfluid ⁴He in a superleak might produce a substantial cooling effect. The calculation is based on the experimental observation [2] that the superfluid density ρ_s depends on V_R , the relative velocity of the normal and superfluid components. Using the experimental result [3] $\delta \rho_s / \rho_s \approx 1.5\%$ when $V_R \approx 40$ cm sec⁻¹ the predicted temperature change from an initial temperature of 1°K is $\Delta T \approx -0.25$ °K. We have performed a different calculation based on a theoretical expression [3] for $S(T, V_R)$ which predicts a much smaller ΔT . In this letter we discuss both calculations and an experiment which shows that $\Delta T < 4 \times 10^{-5}$ °K.

Both calculations are based on the fact that in a quasistatic adiabatic process the entropy is a constant. For such a reversible process ref. [1] treats the entropy S as a function of T and ρ_s and arrives at the following estimate of the temperature change:

$$\frac{\Delta T}{T} \approx \frac{S_{\lambda}}{C_{V}} \left(\frac{\delta \rho_{s}}{\rho_{s}} \right) \qquad T \lesssim 1^{\circ} \mathrm{K}$$
(1)

where S_{λ} is the entropy at lambda point, and C_{V} is the specific heat. Using the experimental value [3] of $\delta \rho_{\rm s} / \rho_{\rm s}$ one gets $\Delta T \approx -0.25^{\circ}$ K at $T \approx 1^{\circ}$ K. This estimate is not valid because eq. (1) was derived for reversible processes whereas the large value of $\delta \rho_{\rm s} / \rho_{\rm s}$ occurs only under highly irreversible circumstances [2]. In our experiment, described below, we find $\Delta T/T < 4 \times 10^{-5^{\circ}}$ K.

Our calculation of the temperature change starts from an expression [4] for the entropy

$$S(T, V_{\rm R}) \approx S_{\rm o}(T) + \frac{1}{2} V_{\rm R}^2 \frac{\partial}{\partial T} \left(\frac{\rho_{\rm n}}{\rho} \right)$$
 (2)

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The equation for a reversible change is

$$0 = dS = \left(\frac{\partial S_{o}}{\partial T}\right)_{V_{R}} dT + \frac{1}{2} V_{R}^{2} \frac{\partial}{\partial T} \left[\frac{\partial}{\partial T} \left(\frac{\rho_{n}}{\rho}\right)\right]_{V_{R}} dT$$
$$+ V_{R} \frac{\partial}{\partial T} \left(\frac{\rho_{n}}{\rho}\right) dV_{R} + \frac{1}{2} V_{R}^{2} \left[\frac{\partial}{\partial V_{R}} \frac{\partial}{\partial T} \left(\frac{\rho_{n}}{\rho}\right)\right]_{T} dV_{R}. (3)$$

Since $\rho_n(T, V_R)$ and $\rho(T, V_R)$ are known, the terms on the right hand side of (3) can be computed, and we find that terms proportional to V_R^2 are negligible, giving

$$\mathrm{d}T \approx -\frac{TV_{\mathrm{R}}}{C_{V}} \frac{\partial}{\partial T} \left(\frac{\rho_{\mathrm{n}}}{\rho}\right) \mathrm{d}V_{\mathrm{R}}.$$
(4)

This equation is valid only for flow states which can be achieved reversibly (i.e., without the formation of metastable vortices). It is known [5] that in an apparatus such as ours the flow is reversible for $V_R \approx 4$ cm sec⁻¹. However inserting $V_R \approx 40$ cm sec⁻¹ in eq. (4) provides an upper limit of the rotational cooling at that speed, $|\Delta T| \approx 4 \times 10^{-6^{\circ}}$ K.

Our apparatus consists of a powder cell suspended by plastic rods inside a vacuum can, which is surrounded by a helium bath. The cylindrical brass cell has a 5.08 cm i. d. and 3.33 cm inner height. It is packed with powder (1000-2000Å particle size) using a hydraulic press until 59% of its volume is filled. The cell is soldered closed, and has a 0.02 cm i.d. stainless steel capillary through the top for filling. The three plastic rods which support the cell inside the vacuum can are 0.32 cm diameter and 524 cm long. A carbon resistance thermometer is varnished into a small copper block soldered to the bottom of the powder-cell. The leads are heat sunk to the bath, along with the capillary, by means of a thick copper wire attached to the top of the vacuum can.

During each rotation the bath temperature was regulated [7], however, the frictional heating of bearings in the bath caused the temperature to rise by 6 mK. The termal isolation provided by the vacuum can prevented this heating from interfering with measurements. The isolation was checked by rapidly changing the regulated bath temperature by an amount ΔT while monitoring the cell temperature. The time τ_1 for the cell temperature to change by $\frac{1}{2} \Delta T$ was ≈ 40 s. For this experiment it is only necessary that the time τ_1 for the cell to reach thermal equilibrium with the bath be much longer than the time τ_2 required for the cell to reach internal equilibrium. The time for the cell to reach internal thermal equilibrium is very short due to the high thermal conductivity of the superfluid. The time for it to reach flow equilibrium is the time it takes to bring the normal fluid into rotation. A calculation [8] shows this

time is $\approx 10^{-8}$ s., so $\tau_1 \ge \tau_2$. At the beginning of a run the bath is pumped to a temperature below 4°K while exchange gas is in the vacuum can. Then He gas is passed through a purifying cold trap and condensed via the capillary until the powder cell was slightly over-filled. Then the vacuum space was pumped out.

While monitoring the temperature the cell is brought up to a peak speed of 23.6 radians per second by a constant acceleration applied for 15 s. By examining the reported behavior of a similar system [5] we estimate that in our experiment the maximum $V_{\rm R} \approx 30$ cm sec⁻¹ With a temperature resolution* of 2×10^{-5} °K no cooling is observed upon acceleration.

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* The time constant of the thermometer electronics was 0.3 s.

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