Nuclear Anapole Moment

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Berkeley, 2006

Plan of the talk

Weak Interactions in Atoms

Charged and Neutral Currents. Effective P-odd Hamiltonian

Nuclear Anapole Moment

Analytical model of Flambaum & Khriplovich

Weak Coupling Constants

What Anapole Moments can Give to the Theory

Outline

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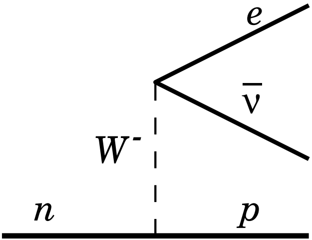
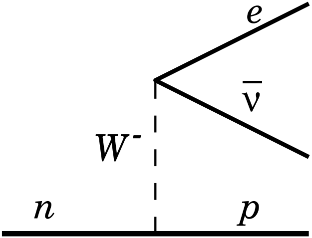
Weak currents



Charged currents can be seen in nuclear decays and other inelastic processes.

Neutral currents can be also seen in elastic scattering. **In atomic physics they lead to additional non-Coulomb interaction of the electrons with the nucleus and with each other.**

Weak currents



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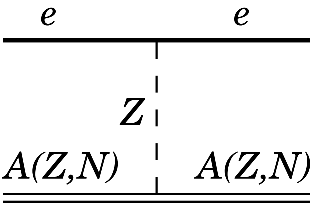

Neutral currents can be also seen in elastic scattering. **In atomic physics they lead to additional non-Coulomb interaction of the electrons with the nucleus and with each other.**

Weak neutral currents

- Because of the very large mass of Z-boson, ∼100 GeV, the weak interaction is contact on atomic scale.
- It includes P-even and P-odd (PNC) parts.
- P-even part leads to small corrections to isotope shift and to hyperfine structure.
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Effective *P*-odd electron-nucleus interaction

$$\begin{aligned} H_P &= H_P^{\mathrm{nsi}} + H_P^{\mathrm{nsd}} \\ &= \frac{G_{\mathrm{F}}}{\sqrt{2}}\Big(-\frac{Q_{\mathrm{W}}}{2}\gamma_5 + \frac{\kappa}{I}\,\gamma_0\vec{\gamma}\vec{I}\Big)\rho(\vec{r}), \end{aligned}$$

where $G_F \approx 1.2225 \times 10^{-14}$ a.u. is the Fermi constant, $\vec{I}$ is nuclear spin, $\vec{\gamma}$ are Dirac matrices, and $\rho(\vec{r})$ is nuclear density. Dimensionless constants Q_{W} and κ characterize the strength of the NSI and NSD parts respectively.

Weak charge

In the lowest order the standard model gives:

$$Q_{\mathrm{W}} = -N + Z(1 - 4\sin^2\theta_{\mathrm{W}}) \approx -N,$$

where N is the number of neutrons and θ_{W} is Weinberg angle. Radiative corrections to this expression change Q_{W} by few percent:

$$Q_{\mathrm{W}} = -0.9857\,N + 0.0675\,Z.$$

NSD coupling constant κ

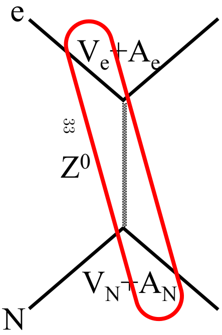
There are three contributions to the constant κ in H_P^{nsd}:

$$\begin{aligned} \kappa &= \frac{K}{i+1}\kappa_A + \kappa_2 + \kappa_{Q_w}, \\ K &\equiv (-1)^{i+1/2-l}(i+1/2). \end{aligned}$$

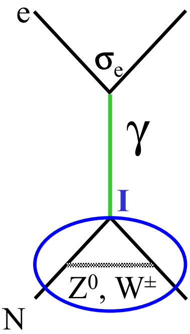
where κ_A is the anapole moment constant, κ_2 corresponds to the weak neutral currents, and κ_{Q_w} appears as a radiative correction to the NSI part; i is nuclear spin and l is orbital angular momentum of the valence nucleon.

Typically $|Q_w| \sim 100|\kappa|$.

V_eA_N Weak Neutral Current & Anapole Moment



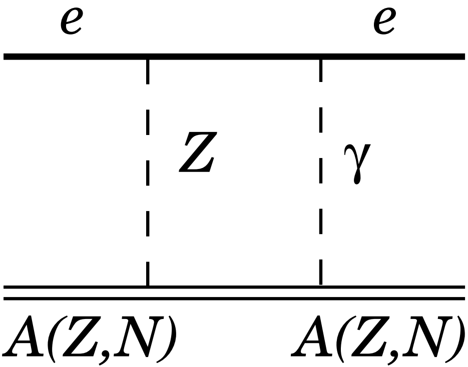
(a)
Weak Neutral Current.



(b)
Anapole moment.

Hyperfine correction to the Weak Neutral Current

(second order radiative correction to the weak amplitude)



$V_e A_N$ Weak Coupling Constant

In the nuclear shell model

$$\kappa_2 = \frac{1/2 - K}{i+1} C_{2\nu},$$

where $C_{2\nu}$ is coupling constant for the valence nucleon:

$$C_{2n} \approx -C_{2p} \approx \frac{\lambda}{2}(1 - 4\sin^2\theta_W),$$

and $\lambda \equiv g_A/g_V \approx 1.25$.

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Nuclear Toroidal Current

- In the nonrelativistic approximation PNC interaction of the valence nucleon with the nuclear core has the form:

  $$H_P \sim \frac{G_F g_n}{2\sqrt{2}c} \frac{(\vec{\sigma}\vec{p})}{m_p c} n(r),$$

  where $n(r)$ is core density and g_n dimensionless effective weak coupling for valence nucleon.
- As a result, the spin $\vec{\sigma}$ acquires projection on the momentum $\vec{p}$ and forms spin spiral.
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Anapole constant κ_A

In 1980 Flambaum & Khriplovich have shown that in the nuclear shell model

$$\kappa_A \approx 1.15 \cdot 10^{-3} A^{2/3} \mu_n g_n,$$

where $A = Z + N$ is the number of nucleons; μ_n and g_n are magnetic moment in nuclear magneton and weak coupling constant of the unpaired nucleon.
For nuclei with unpaired proton and neutron we have:

$$\begin{aligned} \mu_p &= \ \ 2.8\mu_N, & g_p &\approx \ \ 5; \\ \mu_n &= -1.9\mu_N, & g_n &\approx -1. \end{aligned}$$

⇒ The anapole moment is much bigger for nuclei with unpaired proton.

Estimates of Anapole Moments of Some Nuclei

(assumed couplings: $g_p = 4$ and $g_n = -1$; $\kappa'_A = \frac{K}{i+1}\kappa_A$)

Nucleus	i	l	$100 \times \kappa'_A$	$100 \times \kappa_2$	$\lvert\kappa_a/\kappa_2\rvert$
valence neutron					
$^{87}\mathrm{Sr}_{38}$	$9/2$	4	-3.9	5.0	0.8
$^{137}\mathrm{Ba}_{56}$	$3/2$	2	4.6	-3.0	1.5
$^{173}\mathrm{Yb}_{70}$	$5/2$	3	4.5	-3.6	1.3
$^{199}\mathrm{Hg}_{80}$	$1/2$	1	5.0	-1.7	2.9
$^{201}\mathrm{Hg}_{80}$	$3/2$	1	-6.0	5.0	1.2
valence proton					
$^{27}\mathrm{Al}_{13}$	$5/2$	2	-10.0	-5.0	2.0
$^{69}\mathrm{Ga}_{31}$	$3/2$	1	-17.0	-5.0	3.4
$^{81}\mathrm{Br}_{35}$	$3/2$	1	-19.0	-5.0	3.8
$^{115}\mathrm{In}_{49}$	$9/2$	4	-27.0	-5.0	5.2

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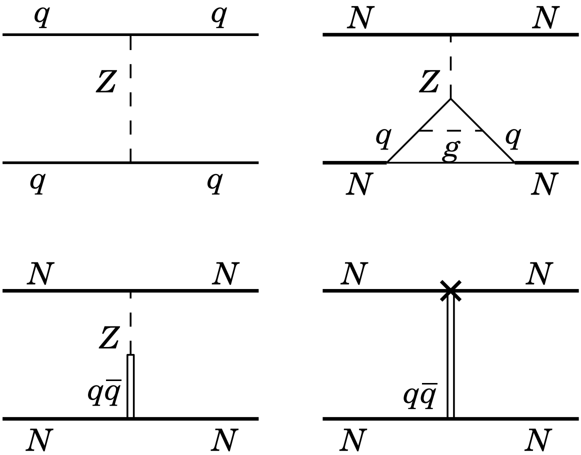
Weak interactions inside the nucleus

(why do we need many weak coupling constants)



DDH constants

(connection to couplings g_p & g_n)

There are 7 independent weak couplings for π-, ρ-, and ω-mesons known as DDH constants.
Proton and neutron couplings can be expressed in terms of 2 combinations of these constants:

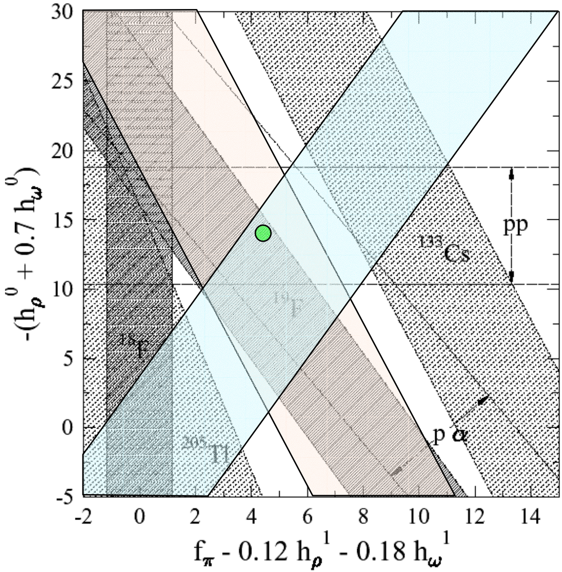
$$\begin{aligned} g_p &= 8.0 \times 10^4 \left[70\tilde{f}_\pi - 19.5\tilde{h}^0 \right], \\ g_n &= 8.0 \times 10^4 \left[-47\tilde{f}_\pi - 18.9\,\tilde{h}^0 \right], \end{aligned}$$

where

$$\begin{aligned} \tilde{f}_\pi &\equiv f_\pi - 0.12h^1_\rho - 0.18h^1_\omega, \\ \tilde{h}^0 &\equiv h^0_\rho + 0.7h^0_\omega. \end{aligned}$$

Experimental data for DDH constants

(Haxton & Wieman, 2001)



Conclusions

- At present the data for weak nuclear constant is inconsistent. That may indicate the problems both with the theory and with the experiment.
- AM of the nuclei can give very important information, which can help to better understand nuclear weak interactions.
- AM of the nuclei with unpaired neutron are particularly interesting, as they depend on the different combination of DDH constants compared to most other experiments.
- For the nuclei with unpaired neutron $\kappa_A \approx -\kappa_2$.
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