Why Can Bose-Einstein Condensates Interfere?

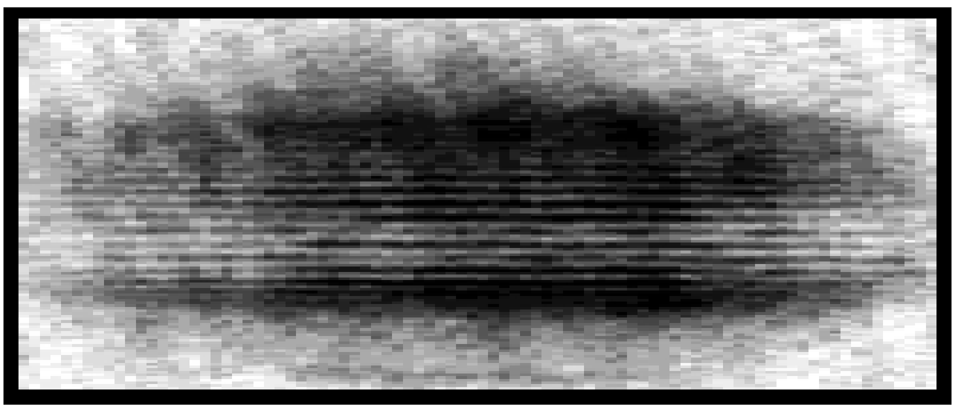

Figure ruthlessly stolen from W. Ketterle's Web Page at http://rleweb.mit.edu/rlestaff/p-kett.htm

Outline of Talk

- Canonical relation between phase and number
- A brief historical tangent
- What all the fuss was about
- The theoretical solution
- The experimental proof
- The end

Canonical relation between phase and number

Consider a collection fo bosons in a harmonic oscillator potential in a mode with frequency ω

$$E = n\hbar\omega$$

bosons

$$\Delta E = \Delta n\, \hbar\omega$$

Heisenberg Says:

$$\Delta E\, \Delta t = \Delta n\, \hbar\omega \Delta t = \Delta n\, \hbar\, \Delta\phi \geqslant \hbar$$

$$\Delta n\, \Delta\phi \geqslant 1$$

A brief historical tangent

$$N = a^{+}a$$

Dirac suggested in 1927 a possible "phase operator:"

$$e^{i\hat{\phi}} = a\,\hat{N}^{-1/2}$$

So that

$$[\hat{N}, \hat{\phi}] = i$$

↓

$$\Delta n\, \Delta\phi \geq ½$$

In 1962, Peter Carruthers assigned the analysis of Dirac's phase operator as a problem in an advanced quantum mechanics class in order to get his students to do the work for him. He offered a "bonus" that turned out to be a single Budweiser Beer.

See M.M. Nieto, "Quantum Phase and Phase Operators: Some Physics and Some History," *in Physica Scripta,* vol, T48 1993
(QC 174.85 P48 Q36 1993 PHYS)

What all the fuss was about?

- ”Problem” is than in a BEC, you can in principle determine without doubt how many atoms you have. That would mean that the phase of a BEC is meaningless.

- Same “problem” plagued ultra-low people.

- Kagen *et. al.* predicted that at first BEC would be a “semi-condensate” with long-range order (phase) occuring on a longer time scale

  Bose Einstein Condensation, ed. A Griffin *et. al.*, 1995, p. 202

- Stoof predicted that a coherent BEC would form immediately.

  Bose Einstein Condensation, ed. A Griffin *et. al.*, 1995, p. 226

The theoretical solution

S. M. Barnett offered a solution:

> "Our simple idea can be summed up in one sentence. *A Bose-Einstein condensate has a preferred phase because it is meaningful, on a macroscopic timescale, to ascribe one to it."*

S.M. Barnett, K. Burnett, and J.A. Vacarro, "Why a Condensate can be Thought of as having a Definite Phase," Journal of NIST, Vol. 101, Number 4, 1996

- Since the BEC is a quantum system, it's properties depend "in a large part" on the measurements you can perform on it.
- The BEC is an open system. You can gain/lose atoms
- Therefore BEC are note made in pure states. BEC is highly entangled with environment. Use a density state to describe mixed state (typically diagonal in Fock state representation).

What do they do from there?

- They describe in more detail the nature and representation of a BEC from both a Fock state representation and a coherent state representation. They conclude that both are legitimate.

- They then discuss how the density matrix for a damped harmonic oscillator will evolve.

- They then show that number states and coherent states have decay rates given by:
    - $2\, n\, \Gamma$-- rate of changing number state (live until one particle is lost)
    - $\langle n \rangle^{1/2}\, \Gamma$-- rate of leaving a coherent state (can live much longer)
    - Γ-- rate of decay of mean field

- They then find that the spread in phase is like $\frac{1}{2} \langle n \rangle^{-1/2}$

Yvan Castin and Jean Dalibard, "Relative Phase of two Bose-Einstein Condensates," *Phys. Rev. A,* Vol. 55, number 6, June 1997

The experimental proof

Ketterle, W *et. a.*. **Observation of interference between two Bose condensates.** Science, vol.275, (no.5300), American Assoc. Adv. Sci, 31 Jan. 1997. p.637-41.