# A limitation to the analogy between pure electron plasmas and two-dimensional inviscid fluids

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Thermal corrections to $\mathbf{E}\times\mathbf{B}$ drifts cause small structures in pure electron plasmas to smear. This smearing breaks the strict analogy between the behavior of pure electron plasmas and the behavior of two-dimensional inviscid fluids. The causes and consequences of this smearing are discussed, a criterion for the validity of the plasma/fluid analogy is constructed, and experimental examples of the failure of the analogy are presented. The criterion indicates that fluid-like behavior can persist to scale lengths far smaller than the normal "collective" behavior limit.

## I. INTRODUCTION

Electrons confined within a Penning/Malmberg[1,2] trap are often assumed to undergo bounce averaged $\mathbf{E}\times\mathbf{B}$ drift dynamics. However, as electrons bounce at the ends of a finite length pure electron plasma, they are subjected to electric fields that are not present inside the bulk of the plasma.[3,4] These fields cause additional, edge-dependent, $\mathbf{E}\times\mathbf{B}$ drifts. These drifts are energy dependent because electrons with different axial kinetic energies penetrate to different depths at the plasma ends. Consequently, the spread in the electron energies present in any finite temperature plasma leads to "smearing" in the plane transverse to the magnetic field $\mathbf{B}$. This smearing can affect both the structure and dynamics of the plasma.

The equations for the bounce averaged $\mathbf{E}\times\mathbf{B}$ drift motion of a pure electron plasma and the motion of a two-dimensional (2-D), inviscid fluid are isomorphic. Consequently, the dynamics of these systems are identical.[5,6] The analogy between these systems (hereafter called the fluid analogy) has been exploited in several recent experiments to yield results that would have been difficult to obtain with conventional fluids.[7–11] Future experiments will use electron "fluids" to study many-vortex systems and turbulence.

As these experiments become more ambitious, it becomes important to identify the limitations of the fluid analogy, and to determine which of these limitations are critical. The derivation of the fluid analogy[5,6] contains no scale length or density requirements; in particular, the plasma Debye length does not enter the derivation and is not directly relevant. In principle, the plasma could be both very tenuous and have infinitesimal structure and still behave like a fluid. Nonetheless, effects outside the $\mathbf{E}\times\mathbf{B}$ model will limit the applicability of the analogy. For example, fluid-like behavior will not persist to scale lengths comparable to the electron gyroradius. However, due to the high magnetic fields present in most experiments, this limit is unimportant. Fluid-like behavior also will not persist once collisional transport becomes substantial, but since the collisional transport time scale is $10^3$ or $10^4$ times longer than the relevant dynamical time scales, this limit is likewise unimportant. Plasma temperature-independent finite length effects can produce external shears in very short plasmas which can change the dynamics, but these changes remain within the fluid analogy framework.[12] Unlike the effects mentioned above, the edge-dependent thermal drifts discussed in this paper constitute a basic limit to the validity of the fluid analogy.

The plasma trap used in these experiments[10,13] is typical of pure electron plasma traps, and consists of three collimated cylinders. The plasma is confined inside the center cylinder by the combined action of the strong axial magnetic field ($B=1900$ G) and the electrostatic well formed by biasing the two end cylinders. These traps are discussed at length in the literature.[1,2,10,13,14]

## II. EDGE-DEPENDENT DRIFTS

The edge-dependent drifts are caused by the anomalous electric fields present at the plasma edge. Although the drifts exist for all plasmas, we have found expressions for the drifts only in the limiting cases where the ratio of the plasma radius to the Debye length, $R=r_p/\lambda_D$, is large or small compared to unity. When $R\ll 1$, this field arises from the electrostatic well potential. It is easy to show that an electron at radius $r$ drifts azimuthally with the bounce-averaged velocity

$$\mathbf{v}_e(r)=k_<(r,\theta)\left(\frac{c\varepsilon}{eBL_p}\right)\hat{\theta}, \qquad (1)$$

where $L_p$ is the plasma length, $\varepsilon$ is the electron's axial kinetic energy, and $(r,\theta)$ form a polar coordinate system in the plane perpendicular to the magnetic field. The function $k_<(r,\theta)=2\chi_1 r/r_w$, where $r_w$ is the trap wall radius and $\chi_1$ is the first root of the Bessel function $J_0$, depends only on the electron's position in the transverse plane, and is generally near unity. Although Eq. (1) is exact (for $R\ll 1$) only if the plasma is well confined (i.e. its end occurs far from the confining cylinder) the small numerical corrections to $k_<(r,\theta)$ found when relaxing this condition are unimportant to this paper. Equation (1) has been verified by measuring the drift-induced rotation period of an off-center, quasimonoenergetic electron column around the trap axis. Because the plasma is nearly monoenergetic, the entire column rotates together and little smearing occurs.

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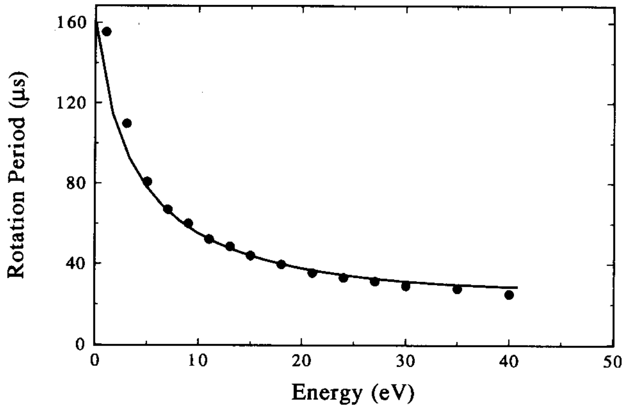


FIG. 1. Measured (points) and predicted (solid line) rotation period vs monoenergetic plasma energy. The plasma length is 17 cm and the trap wall radius is 1.905 cm.

In Fig. 1 we show this period as a function of the electron energy, and we compare the measured period to the theoretical prediction obtained from Eq. (1).

When $R \gg 1$, the edge drifts result from the extra electric fields present in the thin Debye sheaths at the plasma ends.[3,4] Since the sheath thickness is approximately $\lambda_D$, the typical electric field in the sheath is on the order of $E=\phi/\lambda_D=T/e\lambda_D$, where $\phi=T/e$ is the voltage drop across the sheath and $T$ is the plasma temperature. Thus the typical $\mathbf{E}\times\mathbf{B}$ drift velocity in the sheath is $cT/eB\lambda_D$. An electron with energy $\varepsilon$ penetrates to an approximate distance $(\varepsilon/e\phi)\lambda_D$ into the sheath; the fraction of the total time that the electron spends in the sheath is then $(\varepsilon/T)\lambda_D/L_p$. Consequently the bounce-averaged edge drift scales as $c\varepsilon/eBL_p$ times some geometric factor which accounts for the direction of the electric field.[15] The precise calculation, which is too lengthy to be included here, yields[16]

$$\mathbf{v}_e(r,\theta)=k_>(r,\theta)\left(\frac{c\varepsilon}{eBL_p}\right)\hat{\mathbf{s}}, \tag{2}$$

where $\hat{\mathbf{s}}$ is the unit vector in the direction along lines of constant $z(r,\theta)$. In a plasma with a circularly symmetric 2-D profile, $\hat{\mathbf{s}}$ points azimuthally. The geometric factor $k_>(r,\theta)=4\tan\theta_e(r,\theta)$, generally near unity, depends on the angle $\theta_e$ that the plasma's end-shape makes with the plane perpendicular to the magnetic field.[17] Note that the magnitude and scaling of the two expressions for the drifts [Eqs. (1) and (2)] are similar. Consequently, the value for the drift predicted by the equations are likely to be appropriate even in the intermediate region.

## III. FLUID CRITERION

The fluid analogy fails if the magnitude of the edge-drift velocity $v_e$ is larger than the magnitude of the fluid-like drift velocity $v_f$, or if the ratio

$$\Lambda=\frac{v_e}{v_f}=k\,\frac{\lambda_D^2}{\rho L_p} \tag{3}$$

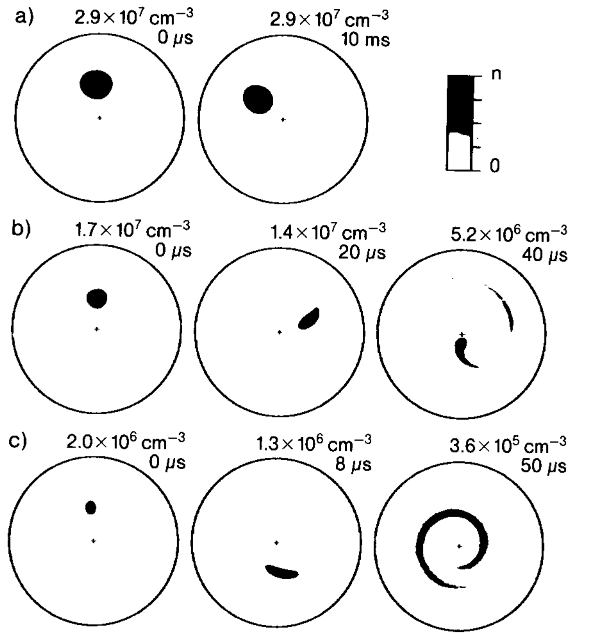


FIG. 2. Three sequences showing the evolution of plasmas with $\Lambda=0.44$ (a), 1.1 (b), and 16 (c). Density is linearly related to the brightness but, to improve the contrast, the brightness has been individually scaled for each image. The maximum density $n$ is shown above each image. The plasmas share the same length ($L_p=1.34$ cm) and temperature ($T=2.4$ eV). Note that the plasma rotates clockwise.

is greater than unity. Here $v_e$ is obtained from Eq. (1) or Eq. (2) and $v_f=2\pi\rho\, nec/B$ is the velocity at the outer edge of a fluid-like vortex of density $n$ and radius $\rho$, and we have used the plasma temperature for the energy $2\varepsilon$. Consequently, the fluid analogy is valid only for features satisfying $\rho>k(\lambda_D^2/L_p)$.

In the definition of $\Lambda$ we use the symbol $\rho$ rather than the symbol $r_p$ to indicate that $\Lambda$ can be generalized to noncircular plasmas. With $\rho$ defined to be the characteristic feature size of a noncircular plasma, Eq. (3) remains dimensionally correct though $k$ may have a different numerical value.

Figure 2 shows the effects of the edge drifts on an off-center column. The column in Fig. 2(a) has $\Lambda=0.44$. This fluid-vortex-like plasma column executes hundreds of revolutions around the device center without any noticeable change. Individual electrons still experience end-drifts; the fluid-like drifts are so strong, however, that electrons that begin to drift out of one side of the column are quickly swept by the fluid drifts around to the other side of the column, where they then drift back into the column. Consequently a threshold, closely related to $\Lambda$, exists below which smearing does not occur and the fluid analogy can be applied confidently. The column shown in Fig. 2(b) has $\Lambda=1.1$. Plasma columns with such intermediate values of $\Lambda$ experience a modest amount of smearing, caused primarily by the loss of cold electrons into a tail. Most of the electrons remain within the central core vortex, and the persistent fluid-like interactions cause the cold tail to wind around the core vortex, while the core vortex itself moves towards the trap center. The smearing eventually ceases

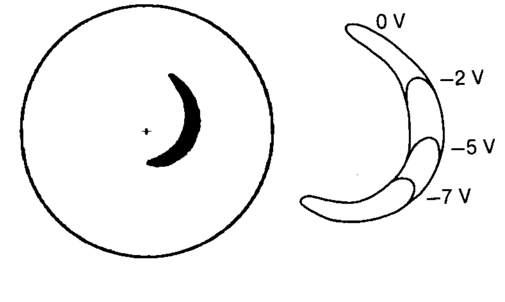


FIG. 3. Image of a plasma for which edge drift is dominant, and an enlarged map of the same plasma with contours showing the images obtained when the confinement gate is raised to the indicated voltages. Density is linearly related to the brightness with maximum brightness corresponding to $1.21\times10^{6}\ \mathrm{cm}^{-2}$. Other parameters are $L_p=1.34$ cm, $T=7$ eV, $\rho=0.142$ cm, and $r=0.76$ cm.

because the edge-dependent drifts decrease as the core approaches the trap center. The column in Fig. 2(c) has $\Lambda=16$. This ratio is so high that the fluid analogy is inapplicable. Edge drift immediately overwhelms any fluid-like electron motion, and the column quickly spreads azimuthally. Although the total charge in Fig. 2(c) is conserved, the column's charge density profile changes drastically. The plasma density is clearly not conserved by the flow, thereby violating one of the basic tenets of inviscid fluid dynamics. Consequently the dynamics shown in Fig 2(c) could not result from temperature *independent* external shears derived from end effects.[12]

Our claim that the drifts are energy dependent is verified by Fig. 3, where we show several images of identically smeared identical plasmas ($\Lambda\gg1$). Each successive image was obtained by bringing the confinement gate to various voltages $V$, each voltage incrementally closer to ground. Because the axial energies of the electrons in the plasma are large compared to the potential variation within the column, each image contains only those electrons whose energies are larger than $eV$. The lower end of the plasma is clearly hotter than the upper end.

In order to analyze the smearing quantitatively, we define the "spread" to be the change, with time, of the distance between the point with maximum density and the farthest point with at least 25% of this maximum density. Figure 4 shows how the spread changes when the displacement off-center, temperature, and density are varied. As predicted by Eq. (1), when $\Lambda\gg1$, the spread depends linearly on the displacement, linearly on temperature, but is independent of density. Experimentally, the onset of smearing occurs when $0.75<\Lambda<3.0$.

Edge drift affects systems other than off-center plasma columns. Although the exact numerical constants may vary, we postulate that the condition $\Lambda\ll1$ remains an appropriate criterion for the validity of the fluid analogy. For example, an annular pure electron plasma corresponds to a circular shear layer and is susceptible to the Kelvin–Helmholtz instability.[18,19] This instability is called the diocotron instability in the plasma and beam literature.[2,5,10,20,21] According to the fluid analogy, the instability should grow rapidly. Several experiments, however, have observed abnormally low growth rates or complete suppression of the instability.[8,10,22] We postulate that these discrepancies are partially caused by edge drift, which leads to azimuthal smearing that slows or suppresses instability growth. Figure 5 shows two plasma annuli with identical radial profiles. Both annuli are produced by axially expanding a shorter annulus. This shorter annulus has $\Lambda\approx1.8$, and is anomalously stable. The fluid velocity is calculated at the outer edge of the annulus. The plasma in Fig. 5(a) is produced by adiabatically expanding the shorter annulus. During this expansion, the density and length change by compensating factors, but the parallel temperature decreases substantially. As a result, the ratio $\Lambda$ is reduced to 0.18, and the annulus becomes unstable to the Kelvin–Helmholtz instability. The annulus in Fig.

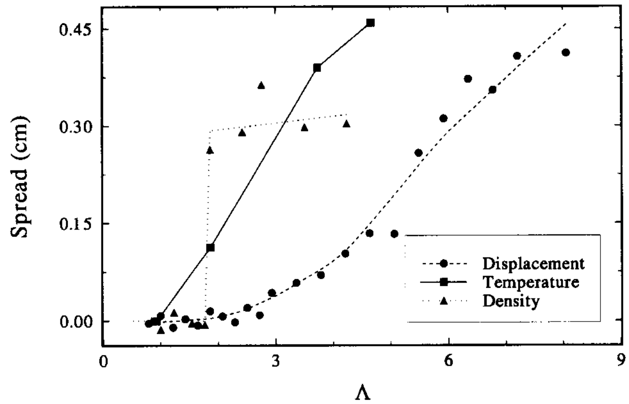


FIG. 4. Spread vs $\Lambda$ for an isolated vortex. The distance off-center is varied for a column with $L_p=1.8$ cm, $T=5$ eV, $\rho=0.16$ cm, and $n=4.5\times10^{6}\ \mathrm{cm}^{-3}$. The temperature is varied for a column with $L_p=12.0$ cm, $\rho=0.16$ cm, $n=1.9\times10^{6}\ \mathrm{cm}^{-3}$, and $r=1.6$ cm. The density is varied for a column with $L_p=1.8$ cm, $T=5$ eV, $\rho=0.16$ cm, and $r=0.76$ cm.

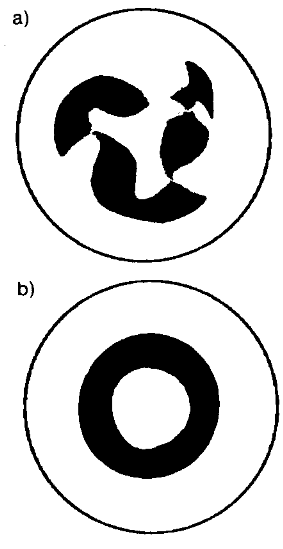


FIG. 5. Adiabatically (a) and nonadiabatically (b) expanded annular plasmas. Density is linearly related to the brightness with maximum brightness corresponding to $2.0\times10^{6}\ \mathrm{cm}^{-2}$. The annulus length is 16 cm and the parallel temperatures immediately after expansion are approximately 0.23 eV (a) and 2.4 eV (b).

5(b), however, is expanded nonadiabatically, and its parallel temperature does not drop. Consequently $\Lambda$ remains near its initial value of 1.8, and the instability remains largely suppressed.

## IV. CONCLUSION

In conclusion, we have established the existence of electron energy-dependent drifts in finite length pure electron plasmas, and we have shown how these edge drifts limit the validity of the analogy between pure electron plasmas and 2-D inviscid fluids. In any plasma there is some critical length scale, well parametrized by $\Lambda$, for which the fluid-like drift and the edge drift are comparable. This scale length may be substantially smaller than the plasma Debye length. In a complex system with many length scales, longer length scales are well described by the analogy while shorter length scales are not. Consequently, finite length plasmas experience density smearing at short length scales. Such smearing is forbidden in idealized inviscid fluid systems. While edge drifts are not strictly analogous to fluid viscosity, both processes reduce small scale structure.

The edge-dependent drifts described in the paper may be important to several systems presently being studied. For example, anomalous results in vortex merger experiments have been attributed to temperature effects.[23] Edge drifts may also be responsible for the observed "viscous" damping of the $\ell=1$ diocotron mode,[24] and may play an important role in the decay of azimuthal turbulence.[25] Finally, edge drifts may be responsible for radial transport that results from asymmetric electric or magnetic confinement fields.

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