# Steering in bicycles and motorcycles

J. Fajans[a)]
*Department of Physics, University of California, Berkeley, Berkeley, California 94720-7300*

(Received 12 April 1999; accepted 12 November 1999)

Steering a motorcycle or bicycle is counterintuitive; to turn *right*, you must steer *left* initially, and vice versa. You can execute this initially counter-directed turn by turning the handlebars explicitly (called counter-steering) or by throwing your hips to the side. Contrary to common belief, gyroscopic forces play only a limited role in balancing and steering [D. E. H. Jones, Phys. Today **23** (4), 34–40 (1970)]. 

Centrifugal forces will throw your bike over on its side if you steer the handlebars in the direction of a desired turn without first leaning the bike into the turn. Indeed, bicycle crashes are often caused by road obstacles like railroad tracks or sewer grates turning the front wheel and handlebars abruptly. Leaning the bike into the turn allows gravitational forces to balance the centrifugal forces, leading to a controlled and stable turn. Thus steering a bike involves a complicated interaction between centrifugal and gravitational forces, and torques applied to the handlebars, all mediated by the bike geometry. (I will use the word bike to refer to both motorcycles and bicycles.)

One method of establishing the proper lean is counter-steering, i.e., explicitly turning the handlebars counter to the desired turn, thereby generating a centrifugal torque which leans the bike appropriately. Counter-steering is employed by both motorcyclists and bicyclists, though most bicyclists counter-steer unconsciously. You may have noticed, however, that while on a bicycle, it is surprisingly difficult to ride clear of a nearby high curb or sharp drop. This is because you must steer towards the edge to get away from the edge. It is easy to directly demonstrate counter-steering on a bicycle. While riding at a brisk pace (possibly downhill to avoid the complications of peddling), let go with your left hand while pushing the right handlebar with the open palm of your right hand. Since your hand is open, you can only turn the handlebar left, but the bike will turn right.

The process of making a counter-steered right turn is illustrated in Fig. 1. (In this description, right and left are in the frame of the rider.) The turn can be broken into five somewhat arbitrarily divided steps:

(a) You initiate the turn by applying a torque to the handlebars, steering the front wheel to the left.

(b) The wheel steers to the left. The rate at which the steering angle increases is set primarily by the moment of inertia $I_s$ of the wheel, fork, and handlebars around the steering axis, and by the ''trail'' (described later.) As the bike is now turning to the left, a centrifugal torque leans both you and the bike frame to the right. Gyroscopic action also leans the bike to the right, but, as I will show later, its effect is negligible.

(c) Transmitted by the fork, the increasing lean attempts to lean the front wheel over as well. For the first time, gyroscopic action becomes important, as the wheel responds to this ''leaning'' torque by attempting to steer to the right, thus counteracting the steering torque. The steering angle stops increasing.

(d) The leaning torque overcomes the steering torque and the wheel steering angle decreases. Note that the lean continues to increase because the bike is still turning left.

(e) As the bike has now acquired substantial leaning velocity, the lean increase cannot end instantly. Driven by the still increasing lean, the wheel steering angle passes smoothly through zero and then points right. The centrifugal torques reverse direction, eventually halting the lean increase and balancing the gravitational torques. As no more leaning torque is applied to the wheel, the steering angle stabilizes, and the bike executes the desired right turn.

Alternately, the required lean can be generated by throwing your hips in the direction counter to the turn. Throwing your hips is how a bike is steered no-hands. The sign of the effect is subtle, but a half-hour session in an empty parking lot should convince you that while riding no-handed, you steer the bike by leaning your shoulders in the direction of the desired turn. Since angular momentum is conserved by a sudden shift of your shoulders, your hips move the opposite way, thereby leaning the bike the opposite way as well. With the bike now leaning, the bike's ''trail'' becomes important. As the steering axis is not vertical, the point of contact of the wheel with the road ''trails'' the intersection of the steering axis with the road [see Fig. 1(a)]. The trail makes the bike self-steer: when the bike leans to the left, the front wheel steers left; when the bike leans to the right, the front wheel steers right. This effect is easily demonstrated by standing beside a bicycle and leaning it from side to side. (The trail is the single most important geometric parameter which enters into the handling of a bicycle.)

The complete hip-turn sequence is similar to the counter-steer sequence illustrated in Fig. 1, except that you initiate the turn by throwing your hips left. The bike leans left as well, and the trail steers the wheel left. Centrifugal torques then lean the center of mass to the right, and gyroscopic forces eventually steer the wheel right.

A mathematical model is necessary to be more precise. As the geometry is complex and the constraints nonholonomic, an exact model is very complicated. I will use a simplified model, good for small leans and steering angles, and will ignore some of the details. Since similar models have been reported before,[1,2] I will only sketch the derivation.

Equating the time derivative of the vertical angular momentum to the torque applied to the handlebars $N_s$ gives

$$\omega I_0 \dot{\lambda} + I_s \ddot{\sigma} = N_s - \frac{Mgb\Delta}{L}\lambda - \frac{Mbv^2\Delta}{L^2}\sigma, \qquad (1)$$

  

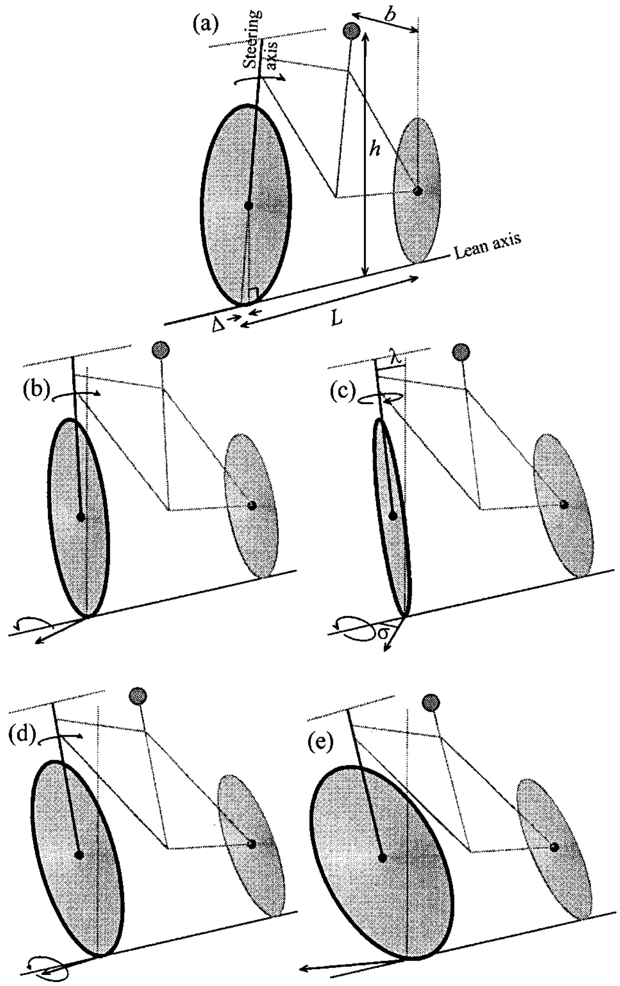


Fig. 1. A counter-steered right turn, as described in the text. The bike geometry is shown in (a) and (c). The center of mass is represented by the filled circle at the location of the seat. The arcs around the steering axis and the lean axis show the direction and approximate magnitude of the torque applied to the handlebars and the net leaning torque.

where $\lambda$ and $\sigma$ are the lean and steering angles, $\omega$ is the wheel's angular rotation frequency, $I_0$ is the moment of inertia of the wheel around its rotation axis, $M$ is the total mass, $g$ is the acceleration due to gravity, $b$ is the horizontal distance from the rear wheel to the center of mass, $\Delta$ is the trail, $L$ is the wheelbase, and $v$ is the velocity of the bike [see Figs. 1(a) and 1(c)]. The first term on the left-hand-side (LHS) comes from changes in the direction of the rotational angular momentum $L_\omega$, and the second term comes from the angular momentum around the steering axis. The second and third terms on the right-hand-side (RHS) come from the trail. The second term is responsible for the wheel steering towards the lean, as described above. The third term attempts to straighten the wheel at high velocity, and comes from a castering effect. Good intuitive derivations of the trail terms are given in Ref. 2.

Considering the wheel only, equating the time derivative of the angular momentum around the lean axis to the torque $N_f$ exerted on the wheel by the fork gives

$$-\omega I_0\dot{\sigma} - \frac{v\omega I_0}{L}\sigma + I_{\lambda w}\ddot{\lambda} = N_f, \tag{2}$$

where $I_{\lambda w}$ is the moment of inertia of the wheel, fork, and handlebars around the lean axis. The first term on the LHS comes from changes in the direction of $L_\omega$ due to changes in the steering angle, and the second from changes in the direction of $L_\omega$ as the bike goes around in a circle of radius $\rho = L/\sigma$. The third term on the LHS comes from the angular momentum of the wheel around the lean axis.

The wheel applies an equal and opposite torque, $-N_f$, to the center of mass through the fork and frame. Equating the time derivative around the lean axis to the torques yields

$$I_\lambda\ddot{\lambda} = -N_f + \frac{hMv^2}{L}\sigma + hMg\lambda + \frac{v\omega I_0}{L}\sigma + \frac{hbMv}{L}\dot{\sigma}, \tag{3}$$

where $h$ is the distance between the lean axis and the center of mass, and $I_\lambda$ is the moment of inertia around the lean axis. The second term on the RHS is the centrifugal torque acting on the bike as it travels in a circle of radius $\rho$, the third term is the gravitational torque, and the fourth term is the gyroscopic action of the rear wheel.

The last term on the RHS of Eq. (3) is an unusual fictitious torque whose origin is not obvious in the derivations in Refs. 1 and 2. The origin of this torque, called the kink torque later in this paper, is described in the Appendix.

Equations (1)–(3) are a reasonable, but approximate, model of bike dynamics. Left out are the finite thickness of the tires, friction, effects other than the trail of the steering axis not being precisely vertical, etc. In particular, an effect due to the deformation of the tire into a cone-like shape is often thought to be important.[3,4] As mentioned above, the equations are valid only for small $\sigma$ and $\lambda$. (Some of the angles in subsequent figures are sufficiently large that higher order terms are needed for fully accurate solutions.) Recent measurements by Jackson and Dragovan on an instrumented bicycle validate a similar mathematical model.[5] The equations are easy to solve numerically. I used Mathcad's adaptive Runge–Kutta routines;[6] the Mathcad worksheets can be downloaded from my website.[7]

The solutions to Eqs. (1)–(3) exhibit growing oscillations. These oscillations are discussed later in the text, and can be suppressed by adding a damping term, $-\Gamma\dot{\sigma}$, to the RHS of Eq. (1). Physically, this damping could come from passive resistance from the rider's arms on the handlebars, or from active responses from the rider.

As an example, take a bicycle with $h = 1.25\,\mathrm{m}$, $L = 1.0\,\mathrm{m}$, $\Delta = 0.02\,\mathrm{m}$, $b = 0.33\,\mathrm{m}$, $I_0 = 0.095\,\mathrm{kg\,m^2}$, $I_s = 0.079\,\mathrm{kg\,m^2}$, $I_{\lambda w} = 0.84\,\mathrm{kg\,m^2}$, $I_\lambda = 163\,\mathrm{kg\,m^2}$, and $\Gamma = 0.65$ Js. Assume that your mass and the bike mass sum to $M = 100\,\mathrm{kg}$ (Ref. 8) and that you travel at the brisk speed of $v = 7$ m/s. You would begin a typical counter-steered right turn ($\rho = 25\,\mathrm{m}$) by torquing the handlebars left (positive $\sigma$) and end the turn by counter-steering in the opposite direction (torquing the handlebars right.) Figure 2(a) shows the handlebar torque time history used in this example. Other time histories are possible, but for this time history, at least, you would *never* torque the handlebars in the direction of the desired right turn until the end of the turn. Moreover, the torques are very small; you would never apply a force greater than 0.9 N (for handholds 0.5 m apart), the equivalent of a weight of 0.092 kg.

Assuming that the torque follows the curve shown in Fig. 2(a), the lean and steering angle responds as shown in the Fig. 2(b). Figure 2(c) shows the torques that cause the lean.

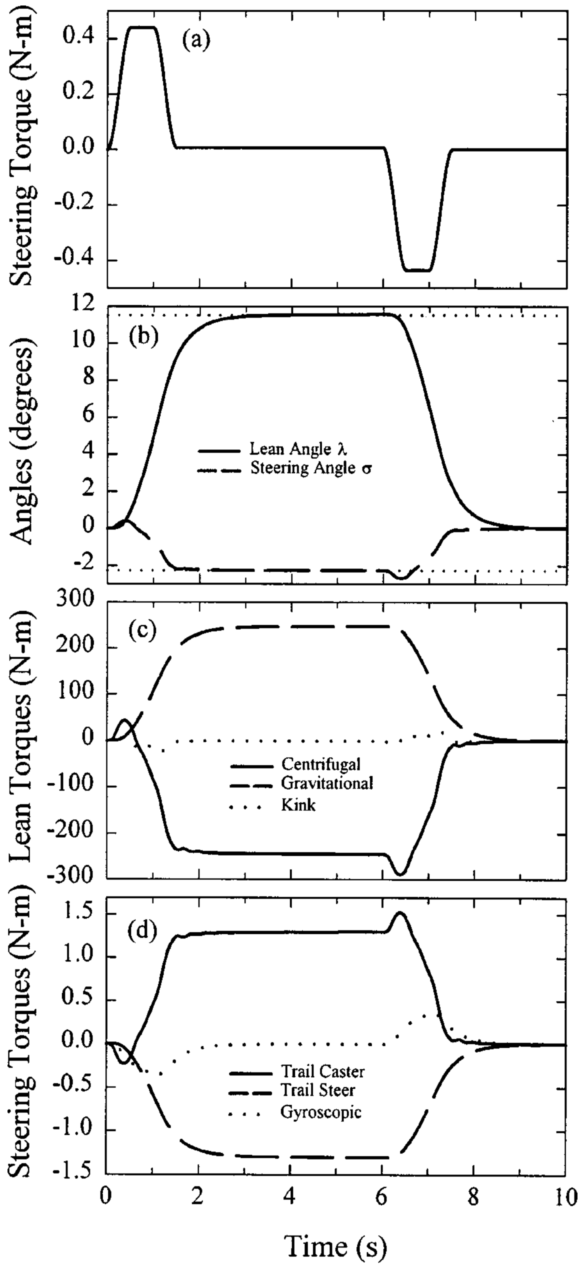


Fig. 2. A counter-steered turn on a bicycle. (a) Torque applied by the rider to the handlebars. (b) Steering $\sigma$ and lean $\lambda$ angles (dotted lines indicate equilibrium angles). (c) Leaning torques. (d) Steering torques.

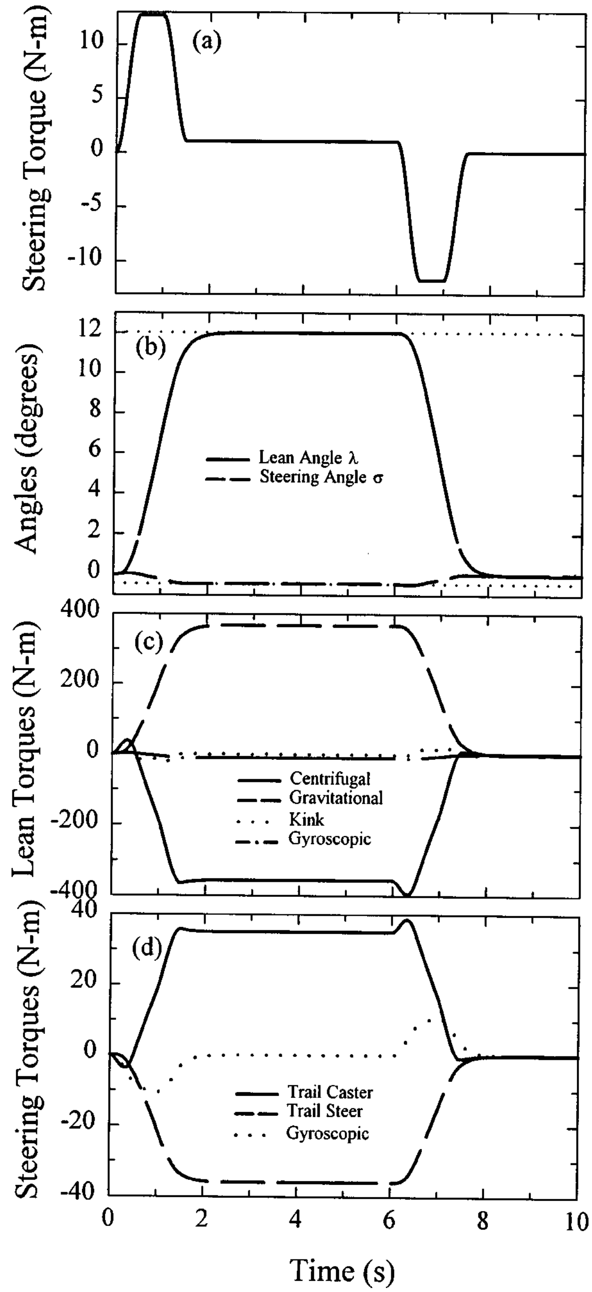


Fig. 3. A counter-steered turn on a motorcycle. (a) Torque applied by the rider to the handlebars. (b) Steering $\sigma$ and lean $\lambda$ angles (dotted lines indicate equilibrium angles). (c) Leaning torques. (d) Steering torques.

The centrifugal torque initiates the lean, with help from the kink torque. The lean angle reaches a steady state when the centrifugal and gravitational torques balance. The contributions from the remaining torques in Eqs. (2) and (3), including all the gyroscopic torques, are not visibly different from zero, and have been omitted from the figure.

Figure 2(d) shows the contributions of the trail torques and of the gyroscopic torque $(-\omega I_0 \dot{\lambda})$ to changing the steering angle [Eq. (1)]. While the gyroscopic torque is non-negligible, it is much smaller than the trail torques and somewhat smaller than the handlebar torque. Thus, in accord with Jones' observation[9] that a bike equipped with a gyro nulling counter-spinning wheel behaves much like a normal bicycle, the ''feel'' of the bike is dominated by the trail.

A counter-steered turn on a motorcycle is qualitatively similar. For a motorcycle with $h=0.60\,\mathrm{m}$, $L=1.54\,\mathrm{m}$, $\Delta=0.117\,\mathrm{m}$, $b=0.77\,\mathrm{m}$, $I_0=0.77\,\mathrm{kg\,m^2}$, $I_s=0.57\,\mathrm{kg\,m^2}$, $I_{\lambda w}=4.0\,\mathrm{kg\,m^2}$, $I_\lambda=118\,\mathrm{kg\,m^2}$, and $\Gamma=3\,\mathrm{Js}$, ridden by a rider whose mass, with the mass of the bike, totals $M=300\,\mathrm{kg}$,[10] at $v=20\,\mathrm{m/s}$, the turn parameters for a sharp[11] right turn ($\rho=200\,\mathrm{m}$) are shown in Fig. 3. The handlebar torques used in this example are higher than for the bicycle: 12.7 N m, or a force of 12.7 N for handholds 0.5 m apart, the equivalent of a weight of 1.3 kg. Contrary to the assertion in Ref. 3, gyroscopic action plays no role in leaning the bike. However, as shown in Fig. 3(d), and in agreement with Ref. 3, it does play a role in steering the front wheel back towards the desired direction.

Scaling relations show why gyroscopic action does not

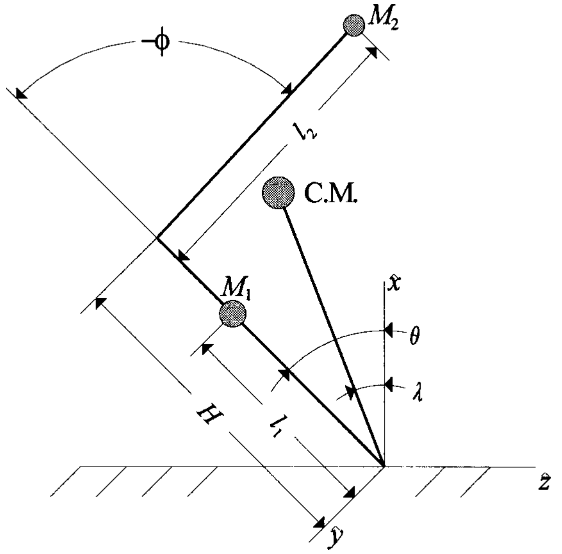

Fig. 4. Hip $\phi$, bike lean $\theta$, and center of mass lean λ angles. Note that in the convention used in this paper, $\phi$ is negative for the positions shown in the figure. As in Fig. 1, this is a head-on view.

contribute to the lean. Dividing the two gyroscopic terms in Eqs. (2) and (3) by the centrifugal term in Eq. (3) gives

$$\frac{2v\,\omega I_0\sigma/L}{hMv^2\sigma/L} \approx \frac{2m}{M}\,\frac{r}{h} \tag{4}$$

and

$$\frac{\omega I_0\dot{\sigma}}{hMv^2\sigma/L} \approx \frac{m}{M}\,\frac{L}{h}\,\frac{1}{\omega\tau}, \tag{5}$$

where $r$ is the wheel radius and $m$ is the effective wheel mass, such that $I_0 = mr^2$. The time $\tau$ is the timescale on which the steering angle changes. Since $r < h$, the first relation is small for any wheel for which $m \ll M$. Likewise $L \approx h$, and $\omega\tau > 1$ for reasonable speeds, so the second relation is also small for $m \ll M$. Increasing the speed only makes $\omega\tau$ bigger, so, for the lean, gyroscopic action actually becomes less important at high speed. (For the parameters of Fig. 2, with $r = 0.33\,\mathrm{m}$, $m = 0.87\,\mathrm{kg}$, and $\tau = 1\,\mathrm{s}$, the ratios are 0.0046 and 0.000 33. For Fig. 3, with $r = 0.29\,\mathrm{m}$, $m = 9.0\,\mathrm{kg}$, and $\tau = 1\,\mathrm{s}$, the ratios are 0.029 and 0.0011, respectively.)

Incorporating hip thrusts complicates the system. For simplicity, I divide the total mass into the bike mass plus lower body mass $M_1$, at distance $l_1$ from the ground, and at angle $\theta$ from vertical, and the upper body mass $M_2$, at distance $l_2$ from the hip pivot point, which itself is a distance $H$ from the ground. Angle $\phi$ is the bend at the hips. The angle λ remains the angle to the center of mass (see Fig. 4). Since changes in $\phi$ are effected by internal forces, such changes conserve the angular momentum around the ground axis. A positive hip thrust $(+\phi)$ leads to a small negative change in the center of mass angle λ. [Actually, the center of mass is almost conserved by hip thrusts, with $\Delta\lambda \approx -(\Delta l^2/2H^2)\Delta\phi$, where $\Delta l$ is some measure of the distance of $M_1$ and $M_2$ from the hip pivot point.]

The only change required to include hip thrusts in Eq. (1) is the substitution of $\theta$ for λ:

$$\omega I_0\dot{\theta} + I_s\ddot{\sigma} = N_s - \frac{Mgb\Delta}{L}\,\theta - \frac{Mbv^2\Delta}{L^2}\,\sigma. \tag{1a}$$

Likewise, Eq. (2) becomes

$$-\omega I_0\dot{\sigma} - \frac{v\,\omega I_0}{L}\,\sigma + I_{\lambda w}\ddot{\theta} = N_f. \tag{2a}$$

Converting Eq. (3) is more complicated. From the position of $M_1$,

$$r_1 = l_1\cos\theta\hat{x} - l_1\sin\theta\hat{z}, \tag{6}$$

the position of $M_2$,

$$r_2 = [H\cos\theta + l_2\cos(\theta+\phi)]\hat{x} - [H\sin\theta + l_2\sin(\theta+\phi)]\hat{z}, \tag{7}$$

and the velocities found by taking the derivatives of these two equations, the angular momentum around the lean $(\hat{y})$ axis is

$$L_y = M_1 l_1^2\dot{\theta} + M_2[H^2\dot{\theta} + l_2^2(\dot{\theta}+\dot{\phi}) + l_2H(2\dot{\theta}+\dot{\phi})\cos\phi]. \tag{8}$$

Equating $\dot{L}_y$ to the external torques yields the new Eq. (3):

$$I_y\ddot{\theta} = -N_f + N_\phi + \frac{hMv^2}{L}\,\sigma + hMg\lambda + \frac{v\,\omega I_0}{L}\,\sigma + \frac{hbMv}{L}\,\dot{\sigma}, \tag{3a}$$

where $M$ is now the total mass, $h$ is the moment arm to the center of mass, $N_\phi$ is the torque that comes from changing the hip angle $\phi$,

$$N_\phi = M_2 l_2 H\sin\phi(2\dot{\phi}\dot{\theta} + \dot{\phi}^2) - M_2 l_2(l_2 + H\cos\phi)\ddot{\phi}, \tag{9}$$

and $I_y$ is the instantaneous moment of inertia around the $\hat{y}$ axis,

$$I_y = M_1 l_1^2 + M_2(H^2 + l_2^2) + 2M_2 l_2 H\cos\phi. \tag{10}$$

Assuming that the hip angle $\phi$ is controlled by the rider and is thus a known function of time, this set of equations is not much more time-consuming to solve than the original set.

The lean and steering angles for a typical no-handed $(N_s = 0)$ right turn on a bicycle are shown in Fig. 5. You would initiate the turn by throwing your hips 21.5 degrees left at $t = 0$ s. At $t = 2$ s you would shift your hips back, realigning the bicycle with your body. The bicycle parameters are the same as those used for the counter-steered bicycle above, with the additional parameters $M_1 = 50\,\mathrm{kg}$, $M_2 = 50\,\mathrm{kg}$, $H = 1.25\,\mathrm{m}$, $l_1 = 1.0\,\mathrm{m}$, and $l_2 = 0.25\,\mathrm{m}$. As before, an examination of the torques (not shown) shows that gyroscopic action plays no role in leaning the bicycle, but does help set the correct steering angle.

As is evident in Fig. 5, the lean, steering, and bike angles exhibit growing oscillations around their equilibrium values. These oscillations were predicted to occur[2] for any passively ridden bike and must be suppressed by more subtle hip motions. Indeed, these oscillations are what make no-handed riding difficult. Normally the oscillations are suppressed by the mass and damping provided by the rider's arms on the handlebars, possibly with some active effort by the rider. As discussed above, this suppression is modeled by the Γ factor.

So far I have analyzed turns made by counter-steering in isolation or hip thrusts in isolation; most turns are made with

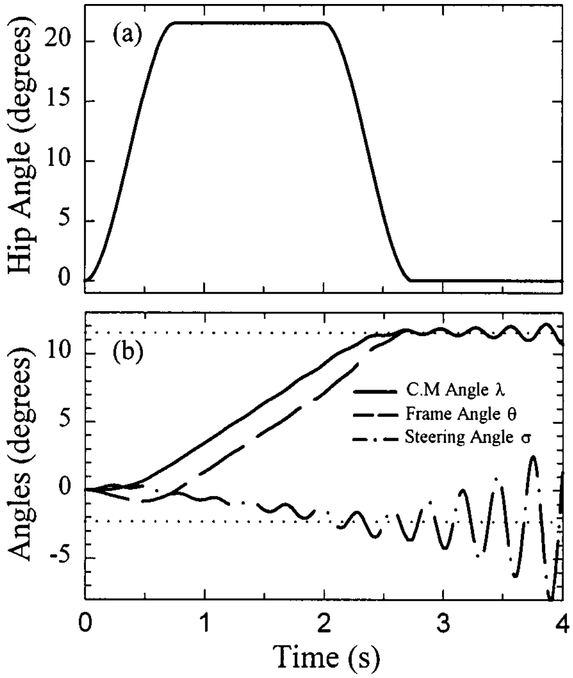


Fig. 5. A no-hands turn executed by hip throwing: (a) hip angle $\phi$ and (b) lean and steering angles. The straight dotted lines in (b) are the equilibrium angles for a turn with $\rho$=25 m at $v=7$ m/s.

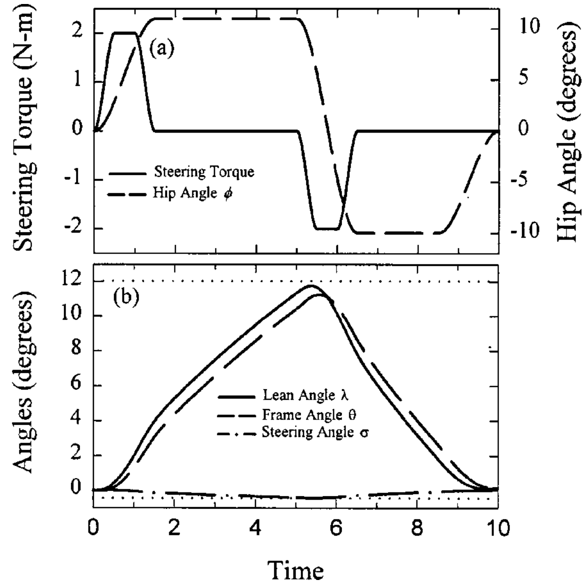


Fig. 7. A right turn executed on a motorcycle by counter-steering and hip throwing: (a) handlebar torque and hip angle and (b) lean and steering angles. The straight dotted lines in (b) are the equilibrium angles for a turn with $\rho$=200 m at $v=20$ m/s. The motorcycle mass is divided up into a mass $M_1=240$ kg at $l_1=0.5$ m, and an upper mass $M_2=60$ kg with $l_2=0.25$ m. The hip pivot point is at $H=0.73$ m, and the other parameters are the same as in Fig. 3.

a combination of the two, i.e., you control both $N_s(t)$ and $\phi(t)$. For example, motorcycle racers shift their weight to the inside while rounding a sharp turn (see Fig. 6). This shift can be loosely modeled as a hip bend, and an example is shown in Fig. 7. One reason you might shift your weight is to reduce the torques applied to the handlebars. For example, the counter-steered turn shown in Fig. 3 requires a maximum torque of 12.7 $\mathrm{kg\,m^2/s^2}$, while the counter-steered plus hip thrust turn shown in Fig. 7 reduces the maximum torque to 2 $\mathrm{kg\,m^2/s^2}$, albeit with a slower entrance to the turn. Just as importantly, you can get into the turn faster while maintaining the same maximum torque. Simulations (not shown) demonstrate a 0.75s advantage for a maximum torque of 12.7 $\mathrm{kg\,m^2/s^2}$ when you bend at the hips by 30 degrees, allowing you to be fully in your turn 15 m sooner than without a hip bend. (On very sharp turns, there is another explanation; shifting your weight to the inside makes the bike less likely to scrape on the road.[12]

On a bicycle, I find that I tend to lean into shallow turns, presumably steering into such turns primarily by hip motion. But for sharp turns I tend to lean away, i.e., I bend so that I am more upright than the bike. Presumably then, I rely on counter-steering for sharp turns. That I often lean away in counter-steered turns is probably a consequence of Newton's third law; counter-steering by pushing on the right handlebar (for a right turn) tends to bend my body to the left. Then again, on some turns I believe I both counter-steer and hip thrust...

Whether with hip thrusts, counter-steering, or both, the bike initially always turns in the direction opposite to the desired turn. However, this initial turn results in a very small incorrectly directed deviation: less than 6 cm in all the examples shown here.

In conclusion, a rider must lean a bike into a turn. Counter-steering and hip thrusts are two common ways of creating the lean, but other ways exist. The rider can take advantage of an uneven road surface, push harder on one pedal than the other, lean the bike over by the handlebars, accelerate with the wheel turned, or employ the growing oscillations shown in Fig. 5. In any event, gyroscopic forces play little role in leaning the bike over, through they do help set the steering angle. The appealing notion that gyroscopic forces are central to bike behavior, often repeated in papers[3,13] and textbooks,[14] is incorrect.

Like many real-world processes, bike steering is a complicated combination of many different actions. Determining what fraction of a turn comes from counter-steering, and

Fig. 6. A motorcycle racer rounding a sharp turn. Note how the rider's mass is shifted off to the side (Ref. 15).

what fraction from hip thrusts, and how these fractions change for different turns, requires a fully instrumented bike that records the bike angle, the steering angle, the steering torques, the velocity, and the hip angle. Jackson and Dragovan's partially instrumented bicycle[5] is an excellent beginning.

## APPENDIX: KINK TORQUE

Equations (2) and (3) are valid in a reference frame rotating at the instantaneous angular frequency $v/\rho=(v/L)\sigma$. As the turning radius changes, this reference frame changes, resulting in the kink torque of Eq. (3). To understand the origin of this torque, consider a bike whose initially straight front wheel is abruptly turned to some angle $\sigma$, as shown in Fig. 8, and whose center of mass is located halfway between the two wheels. Before the front wheel is turned, the front and rear wheel ground-contact points and the center of mass all travel in straight lines in a stationary frame. After the front wheel is turned, both wheel contact points follow arcs. However, while the rear wheel trajectory flows into its arc smoothly, the front wheel trajectory has a kink. Likewise, the trajectory of the point halfway between the wheels has a similar kink. The center of mass, however, continues in a straight line. In the rotating frame, the wheel contact points and center of mass are stationary initially, and the wheel contact points remain stationary after the front wheel is turned. The center of mass, however, appears to be kicked outward, as can be seen by comparing the trajectory of the center of mass to the trajectory of the point half way between the wheels. The two trajectories diverge abruptly due to the kink in the half-way point's trajectory. Superficially this motion is similar to a normal centrifugal force, but the normal centrifugal force has no similar kink. For an abrupt change in the steering angle, creating this kink requires an impulsive force different from the steady-state centrifugal force. Realistically, of course, the steering angle can only be changed gradually, and a gradual change in the steering angle results in the last term[1,2] on the RHS of Eq. (3).

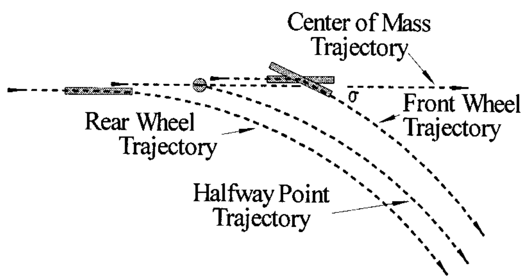


Fig. 8. Trajectories in a stationary frame of the wheels (rectangles), point halfway between the wheels, and center of mass (filled circle), with an abrupt change of the steering angle $\sigma$. Note the kink in the front wheel and halfway point trajectories. For clarity the trajectories are slightly vertically displaced.


[a]Electronic mail: joel@physics.berkeley.edu



[1]S. Timoshenko and D. H. Young, *Advanced Dynamics* (McGraw–Hill, New York, 1948), pp. 239–243.

[2]J. Lowell and H. D. McKell, ''The stability of bicycles,'' Am. J. Phys. **50**, 1106–1112 (1982).

[3]A. J. Cox, ''Angular momentum and motorcycle counter-steering: A discussion and demonstration,'' Am. J. Phys. **66**, 1018–1021 (1998).

[4]J. Robinson, *Motorcycle Tuning:Chassis* (Newnes, Boston, 1994), pp. 1–16.

[5]A. W. Jackson and M. Dragovan, ''An experimental investigation of bicycle dynamics,'' Am. J. Phys. (submitted).

[6]Mathsoft Inc., Cambridge, MA.

[7]http://socrates.berkeley.edu/~fajans.

[8]These parameters correspond to a LeMonde Zurich bicycle. The Zurich has a high end, reasonably aggressive design. The height $h$, which is a function of the rider's position, and the moments $I_s$ and $I_{\lambda w}$ are the most uncertain parameters. The exact value of $I_{\lambda w}$, however, is unimportant because it is much smaller than and acts identically to $I_\lambda$, and the term proportional to $I_s$ is small.

[9]D. E. H. Jones, ''The stability of the bicycle,'' Phys. Today **23** (4), 34–40 (1970).

[10]These parameters are inferred from specifications and pictures on the Harley-Davidson website, and conversations with local motorcycle dealers, for an XLH Sportster 883. See Ref. 8. For a motorcycle, the term proportional to $I_s$ is negligible. The motorcycle's leaning moment $I_\lambda$ is smaller than the bicycle's leaning moment because the center of mass is lower.

[11]The turning radii used in Figs. 2 and 3 are the minimum highway turning radii allowed at the given speeds. See http://www.dot.ca.gov:80/hq/oppd/hdm/chapters/tables/tb203_2.htm.

[12]Adam Curtin, private communication.

[13]J. Higbie, ''The motorcycle as a gyroscope,'' Am. J. Phys. **42**, 701–702 (1974).

[14]W. T. Griffith, *The Physics of Everyday Phenomena* (McGraw–Hill, New York, 1998), pp. 149–150.

[15]Photograph courtesy of Amanda Curtin.


**NO SPECIAL GIFT!**

Carl Seelig, one of Einstein's chief biographers, once wrote to him asking whether he inherited his scientific gift from his father's side and his musical from his mother's. Einstein replied in all sincerity, ''I have no special gift—I am only passionately curious. Thus it is not a question of heredity.''

Banesh Hoffmann, *Albert Einstein—Creator and Rebel* (Penguin Books, New York, 1972), p. 7.