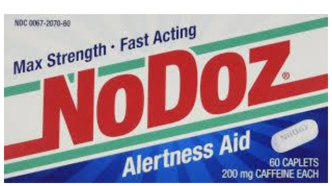
Marty and Orbifolds

Jan de Boer, Amsterdam



Berkeley, March 29, 2019

Date: Thu, 24 Jun 93 15:52:45 PDT
From: halpern%theory.hepnet@Lbl.bitnet
To: deboer@ruunts.fys.ruu.nl
Subject: RE: greetingsfromutrecht

LBL, Particle Theory Group, 24-JUN-1993

Dear Jan,

I look forward to the details of the first-order form.
The C-function paper is NPB357(1991)655.

Regards, Marty

Date: Wed, 8 May 1996 15:56:38 +0200 (CET-DST)
From: HALPERN@vxcern.cern.ch
To: deboer@insti.physics.sunysb.edu
Cc: HALPERN@vxcern.cern.ch
Subject: Re: from CERN

Hi Jan,
I am now here in Toronto, listening to what is probably the worst set of
talks ever conceived. I hope Niels will soon tire of listening to them
so we can get a little work done while we are here.
I presume you got my emails from CERN and London. How is it going?
Best, Marty

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$$J^a(z)J^b(w) = \frac{G^{ab}}{(z-w)^2} + \frac{if_{ab}{}^c J_c(w)}{z-w} + \dots$$

$$T(z) = L^{ab} : J_a J_b :$$

$$L^{ab} = 2L^{ac}G_{cd}L^{db} - L^{cd}L^{ef} f_{ce}{}^a f_{df}{}^b - L^{cd} f_{ce}{}^f f_{df}{}^{(a} L^{b)e}$$

This includes cosets but also lesser known constructs such as nested cosets.

To construct a consistent CFT with one of these stress tensors we need to gauge everything that commutes with it. When such a consistent gauging exists remains an important open problem.

stress tensor	commutant
$T = T_g - T_h$	$\hat{h}$
$T = T_g - T_h + T_k$	$\hat{h}/\hat{k}$
T	$T_g - T, \ldots$

Going back to 1997, in a paper with Borisov and Schweigert, Marty also started working on orbifolds.

The combination of orbifolds with the Virasoso Master Equation potentially leads to *large class* of CFTs/string theories.

The approach to orbifolds, characteristic of Marty, was very much based on algebra.

This approach roughly works as follows:

Suppose H generates automorphisms of some affine Lie algebra g.

Given some hϵH, find linear combinations of the currents so that they have a fixed eigenvalue under the action of h.

In the h-twisted sector, these linear combinations will have a fractional mode expansion, but their OPE is identical to that in the untwisted sector.

The expression for the stress tensor also remains the same

$$T(w) = \oint \frac{dz}{2\pi i} \frac{L^{ab} J_a(z) J_b(w)}{z - w}$$

but this normal ordering is not quite the same as the normal ordering with respect to the fractional mode expansion of the currents. The difference between the two can be used to compute energy of twisted sector ground states.

This idea can be extended to twisted primaries, twisted vertex operators, twisted KZ equations, and also to CFTs with boundaries and orientation-reversing symmetries (not quite orientifolds).

Halpern, Wang
hep-th/0005187

2.6 The cyclic permutation orbifolds $A(\mathbb{Z}_\lambda)/\mathbb{Z}_\lambda$

As an example, we recall the seminal case[19] of the general cyclic permutation orbifold $A(\mathbb{Z}_\lambda)/\mathbb{Z}_\lambda$,

$$a \to aI, \qquad L^{ab} \to L^{aI,bJ} = L^{ab}_{I-J}, \qquad I = 0, ..., \lambda - 1 \tag{2.40a}$$

$$n(r), \mu \to r, aj, \qquad \hat{J}_{n(r)\mu} \to \hat{J}^{(r)}_{aj}, \qquad \chi(\sigma)_{n(r)\mu} \to \chi(\sigma)_{raj} = \sqrt{\rho(\sigma)} \tag{2.40b}$$

$$\mathcal{G}_{raj;sbl}(\sigma) = \rho(\sigma)k\eta_{ab}\delta_{jl}\delta_{r+s,0 \bmod \rho(\sigma)}, \qquad \mathcal{F}_{raj;sbl}{}^{tcm}(\sigma) = f_{ab}{}^c\delta_{jl}\delta_l{}^m\delta_{r+s-t,0 \bmod \rho(\sigma)} \tag{2.40c}$$

$$\hat{J}^{(r)}_{aj}(z)\hat{J}^{(s)}_{bl}(w) = \delta_{jl}\{\frac{\rho(\sigma)k\eta_{ab}\delta_{r+s,0 \bmod \rho(\sigma)}}{(z-w)^2} + \frac{if_{ab}{}^c\hat{J}^{(r+s)}_{cj}(w)}{(z-w)}\} + O((z-w)^0) \tag{2.40d}$$

$$\hat{J}^{(r)}_{aj}(ze^{2\pi i}) = e^{-\frac{2\pi ir}{\rho(\sigma)}}\hat{J}^{(r)}_{aj}(z), \qquad \hat{J}^{(r\pm\rho(\sigma))}_{aj}(z) = \hat{J}^{(r)}_{aj}(z) \tag{2.40e}$$

$$\hat{T}_\sigma(z) = \sum_{r=0}^{\rho(\sigma)-1}\sum_{j,l=0}^{\frac{\lambda}{\rho(\sigma)}-1}\mathcal{L}^{raj;-r,bl}(\sigma) : \hat{J}^{(r)}_{aj}(z)\hat{J}^{(-r)}_{bl}(z) : \tag{2.40f}$$

$$\mathcal{L}^{raj;-r,bl}(\sigma) = \frac{1}{\rho(\sigma)}\sum_{s=0}^{\rho(\sigma)-1} e^{-\frac{2\pi iN(\sigma)rs}{\rho(\sigma)}} L^{ab}_{\frac{\lambda}{\rho(\sigma)}s+j-l} \tag{2.40g}$$

$$\hat{\Delta}_0(\sigma) = \frac{\lambda k\eta_{ab}}{4\rho^2(\sigma)}\{\frac{\rho^2(\sigma)-1}{3}L^{ab}_0 - \sum_{r=1}^{\rho(\sigma)-1} csc^2(\frac{\pi N(\sigma)r}{\rho(\sigma)})\, L^{ab}_{\frac{\lambda}{\rho(\sigma)}r}\} \tag{2.40h}$$

$$a, b = 1, ..., \dim\mathfrak{g}, \qquad \bar{r}, \bar{s} = 0, ..., \rho(\sigma) - 1, \qquad j, l = 0, ..., \frac{\lambda}{\rho(\sigma)} - 1, \qquad \sigma = 0, ..., \lambda - 1 \tag{2.40i}$$

Date: Wed, 8 Mar 2006 17:36:45 -0800
From: Martin B. Halpern <halpern@physics.berkeley.edu>
To: Jan de Boer <jdeboer@science.uva.nl>
Subject: Re: fire

Hi Jan,

Thanks for your concern. I lost nothing, and have temporarily moved my office to the house during repairs.
The damage, due to a heater, was only the top half-surface of the desk, but the fire people did serious damage with water and soot, so it is indeed the end of an era.

More later, Marty

In 2007, Marty wrote 5 papers with the goal to extend the “orbifold program” to string theory.

The idea is roughly to consider the orbifold

$$(26\ \text{bosons} + \text{ghosts})^N / H$$

where H must leave $Q = \sum_{i=1}^{N} Q^i_{\text{BRST}}$ invariant.

One can then consider the BRST cohomology of the orbifold theory to get a physical state condition. Along the way, Marty seems to rediscover the “long string phenomenon” in some examples.

In paper V of the series there are computations of amplitudes

$$\hat{A}_4^{(2)}(\{\mathcal{T}\}) = \delta^{26}\left(\sum_{i=1}^{4} T^{(i)}\right) S^T \mathbb{1}_2 S \times$$
$$\times \int \frac{d^2 z}{8\pi} \, |z|^{-\left(T^{(2)}\cdot T^{(1)}+1\right)} |1-z|^{-T^{(2)}\cdot T^{(3)}} \times$$
$$\times \left(\left|\frac{1-\sqrt{z}}{1+\sqrt{z}}\right|^{-T^{(2)}\cdot T^{(3)}} + \left|\frac{1+\sqrt{z}}{1-\sqrt{z}}\right|^{-T^{(2)}\cdot T^{(3)}} \right) \quad (2.21)$$

and of the cosmological constant

$$\hat{\Lambda}_N^{(\lambda)} = -\frac{1}{2}\int_{F_N} \frac{d^2\tau}{(\operatorname{Im}\tau)^2} \left\{ \lambda Z(\tau,\bar{\tau}) + (\lambda-1)\sum_{r=0}^{\lambda-1} Z\left(\tfrac{\tau+r}{\lambda}, \tfrac{\bar{\tau}+r}{\lambda}\right) \right\} \qquad (5.2)$$

issue

The story continues with 4 more papers in 2010 which contain e.g.

- A more elaborate and general description of the ghosts and physical state conditions in each twisted sector

$f_j(\sigma)$	$\hat{c}_j(\sigma)$	$\hat{a}_{f_j(\sigma)}$
1	26	1
2	52	17/8
3	78	29/9
4	104	69/16
5	130	27/5
6	156	155/24

The second paper 1008.2576 is 64 pages but contains the same paper twice…..??

The third paper associates space-time dimensions and signatures to each of the twisted sectors through an analysis of zero modes.

The last paper arXiv:1010.1893 discusses a class of examples

The Lorentzian Space-Times of the Orientation-Orbifold String Systems

M.B.Halpern*
Department of Physics
University of California
Berkeley, Ca. 94708, USA

October 29, 2018

The last three paragraphs of the last paper read:

We have also included a number of introductory, successful tests of the no-ghost conjecture [1-5] for these theories (see Secs. 10-13).

It seems that the orientation-orbifold string systems studied here are the simplest among the orbifold-string theories of permutation-type [1-8], not least because they exhibit only a single graviton per orbifold and contain ordinary ghost-free $D(\sigma) \leq 26$ - dimensional string subsystems with quantized intercept $a(\sigma) \leq 1$ (see Sec. 13). A next step in this program should be the study of the twisted $\hat{c} = 52$ open-string vertex operators of the orientation-orbifolds and the construction of the open-string sectors at tree level, following the text and Appendix of Ref. 4. Given our historical understanding [24], one expects that these sectors will be the simplest in which to elevate the no-ghost conjecture to a theorem for the new string theories.

We conclude this paper with a final remark on another important direction in the program. On the basis of the enhanced target-space Lorentz symmetries [8] studied here for the bosonic prototypes, we expect correspondingly-enhanced target-space supersymmetries – and non-tachyonic spectra – in the superstring generalizations [1] of the orientation-orbifold string systems.

Is there a way to think about these orbifold string theories?
What is their geometry, what is the relevant moduli space?

We know examples where the usual geometry of Riemann surfaces of string theory is extended:

- Superconformal theories (super Riemann surfaces)
- Higher spin theories (flat SL(N,R) bundles or Hitchin systems – we don't really know)
- Coupling to discrete gauge theories (=orbifolds)

But what about Halpern's String Theories (????????)