Dynamic and Debye shielding and anti-shielding*

C. Hansen, A. B. Reimann, and J. Fajans†
Physics Department, University of California, Berkeley, Berkeley, California 94720-7300

(Received 7 November 1995; accepted 2 February 1996)

While shielding in *collisional* plasmas obeys the standard Debye result, shielding in *collisionless* plasmas is far more complex than commonly believed. For example, a one-dimensional (highly magnetized), immobile-ion plasma can, in some circumstances, *anti-shield* a positive test charge; i.e. the plasma becomes more *positive* in the vicinity of the test charge. When shielding does occur, it results from electrons dynamically trapped in the neighborhood of the test charge. A new theory of collisionless (Dynamic) shielding in one, two and three dimensions is presented here, and is in excellent agreement with experiments in pure electron plasmas. Because the distribution functions found in Dynamic shielding are highly non-Maxwellian in the non-linear regime, collisionless Dynamic shielding can be substantially less efficacious than collisional Debye shielding. © *1996 American Institute of Physics.* [S1070-664X(96)94205-8]

I. INTRODUCTION

One of the most fundamental plasma characteristics is that plasmas shield applied electrostatic perturbations. A test charge placed in a plasma will be surrounded by a cloud of oppositely charged plasma particles, which both shields the remaining plasma from the test charge and lowers the electrostatic potential induced by the test charge. The special case of *collisional*, or Debye shielding, in which the plasma remains Maxwellianized throughout, is well understood.[1] Moreover, it is commonly believed[2] that collisions are not actually necessary for shielding; this point of view is so ubiquitous that the requirement that the plasma be Maxwellianized is often not made explicit.[3–5] *Collisionless* shielding is paradoxical, however. As electrons accelerate towards a positive test charge, flux conservation demands that their density decrease. Consequently, a positive test charge can be surrounded by a net *positively* charged plasma, and both the fields from the test charge and the potential in the vicinity of the test charge can be accentuated. In this case a plasma can actually *anti-shield* a test charge.

That one-dimensional (1-D), collisionless plasmas[6] anti-shield has been recognized in one group of papers[7,8] on collisionless shielding. We present here what we believe is the first direct experimental evidence of this phenomenon.[9] According to this group of papers, two-dimensional (2-D), collisionless plasmas should neither shield nor anti-shield; the field from a test line charge is predicted to be unmodified by a collisionless plasma. Three-dimensional (3-D) plasmas are reputed by this group to shield test charges, albeit somewhat weakly. Other sources[10,11] predict that plasmas of all dimensions will shield, but their results are inadequately justified.[12] All these papers[7,8,10,11] ignore the existence of non-collisional trapping mechanisms: two mechanisms of particular importance are transit-time or adiabatic[13,14] trapping and instantaneous[13] trapping. We will show that these mechanisms cannot be ignored, and that collisionless plasmas will normally shield a test charge,[9] in accordance with common belief and the second group of papers. This conclusion is verified by the pure-electron plasma experiments described at the end of this paper. Finally, a third group of papers analyze the instantaneous application of a test potential,[2,15–17] and predict that 3-D plasmas will shield. As discussed later, however, the results of these papers are limited to small applied potentials and are valid only in the restricted time range $1/\omega_p < t < 1/\omega_b$, where ω_p is the plasma frequency and ω_b is the bounce frequency of electrons trapped in the applied potential.

Since collisionless shielding depends on particle dynamics rather than on particle collisions, we call the process Dynamic shielding. Similar, but more complex phenomena such as double layers, Bernstein, Greene and Kruskal (BGK) modes,[18] virtual cathodes, and sheaths have been studied extensively. Indeed, most of the concepts required to analyze Dynamic shielding have been described before, most notably by Gurevich,[13] who analyzed non-collisional trapping mechanisms and self-consistent potential solutions, but did not consider the problem of shielding. Given all this prior work, it is surprising that adiabatic-collisionless shielding itself has not been properly analyzed before.

II. THEORY

While shielding is often conceived to be the masking of the field of a test charge, a more general description is the partial neutralization of an imposed test potential well. This test well may, for instance, be created by biasing a grid in a neutral plasma or by biasing a confinement electrode in a non-neutral plasma. However the well is created, the plasma responds to the net potential: the sum of the test potential and the plasma potential. Since the plasma potential depends, through changes in the plasma density, on the net potential, the response of the plasma must be found by self-consistently solving for the net potential. We will show that the solution depends on the well time history, and the plasma dimensionality and longitudinal extent.

A. One-dimensional adiabatic theory

Energy conservation requires that the velocity of untrapped electrons, subjected to a positive test potential well,

*Paper 6IA2, Bull. Am. Phys. Soc. **40**, 1775 (1995).
†Invited speaker.

 1070-664X/96/3(5)/1820/7/$10.00

increase according to the dimensionless relation $v^2 = v_0^2 + 2\Phi$, where the normalized velocity v is defined such that $v(kT/m)^{1/2}$ is the velocity inside the well, v_0 is the velocity outside the well similarly normalized, m is the mass of the electron, T is the plasma temperature, $kT\Phi/e$ is the self-consistent well depth, and $-e$ is the charge on an electron. The velocity v_0 is a constant of the motion; since any function of a constant of the motion is a solution of the Vlasov equation, the distribution of electrons in the well is $f(v) = f_0[v_0(v)] = f_0[(v^2 - 2\Phi)^{1/2}]$, where $f_0(v_0)$ is the initial distribution. If the electrons come from an infinite length, collisionless, Maxwellian plasma source, then f_0 is a Maxwellian distribution at temperature T and density n_0. The density of untrapped electrons as a function of the well depth is found by integrating $f(v)$ from the minimum electron velocity inside the well, $(2\Phi)^{1/2}$ to infinity, and is given by

$$n_F(\Phi) = n_0 \exp(\Phi)\operatorname{erfc}(\sqrt{\Phi}) = n_0(1 - 2\sqrt{\Phi/\pi} + \Phi + \ldots), \quad (1)$$

where erfc is the complementary error function. This free density, $n_F(\Phi)$ is the density those electrons untrapped by the potential well. Since $n_F(\Phi)$ is always less than n_0, the free electrons can only anti-shield a positive external perturbation.

The conclusion that the plasma will anti-shield[7,8] ignores the possibility that electrons could be trapped in the well by non-collisional mechanisms, thereby increasing the local plasma density. In particular, the conclusion ignores the question of how the well was created. If the well is created instantaneously, slow electrons that happen to be in the well at the time of its creation will be trapped. If, as is more likely, the well is created adiabatically, the transit-time trapping mechanism described below will trap electrons. In fact, the only way to avoid trapping electrons is for the well to be created *before* electrons are allowed into the vicinity of the well or to use a non-monotonic initial distribution function. Only in these limited circumstances do experiments show that the plasma anti-shields.

Transit-time trapping in an infinite length plasma was first discussed by Gurevich.[13,14] The process is most easily visualized for square test wells, but occurs for any shape well. Consider a slow-moving (initial velocity v_s) electron. Although it gains kinetic energy on entering a slowly-deepening test well, its kinetic energy remains constant inside the well, despite the increasing test well depth. If the well depth increases sufficiently during the transit time, the electron may not have sufficient energy to climb out of the well—the electron is trapped. Further changes in the well depth do not change the kinetic energy; the electron's total energy simply varies proportionally as the well depth is increased. Hence the electron's initial velocity v_s is a constant of the motion, and the distribution of electrons in the well is given by $f_0(v_s)$. Further, if the well depth increases very slowly, only particles with velocity $v_s \approx 0$ will be trapped, so their distribution function reduces to $f_0(0)$ (see Fig. 1). Integrating over the trapped electron distribution function gives the trapped density

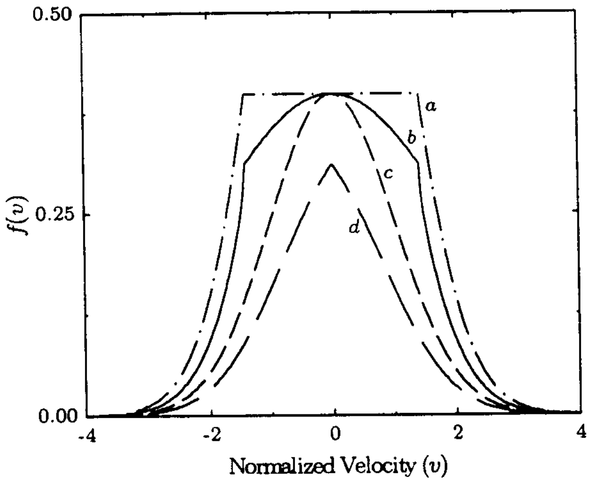

FIG. 1. Self-consistent distribution functions for an applied well voltage of $\Phi_w = 2.5$. Line a is the Debye distribution for an infinite length plasma, line b is Dynamic distribution function for an infinite length plasma, lines c and c' are the Dynamic distributions inside and outside the well for the ratio $L_{in}/L_{out} = 1$, line d is the Instantaneous distribution for an infinite length plasma, and line e is the original Maxwellian distribution before the well is applied (overlaid by line d for small v). Here $C=5$ and the shielding efficacies are 0.86, 0.76, 0.79, and 0.56 for lines a, b, c, and d respectively. Note that line d is likely to be unstable.

$$n_T(\Phi) = 2n_0\sqrt{\Phi/\pi}. \quad (2)$$

The density of trapped electrons increases proportional to $\sqrt{\Phi}$, while, to lowest order, the density of free electrons decreases proportional to $\sqrt{\Phi}$. When the densities of these two classes of electrons are added together to form the total density, the square root terms cancel, leaving a linear first order term[13,14]

$$n_{tot}(\Phi) = n_0 \exp(\Phi)\operatorname{erfc}(\sqrt{\Phi}) + 2n_0\sqrt{\Phi/\pi} = n_0(1 + \Phi + \ldots). \quad (3)$$

To first order this density equals that found when collisions are able to relax the plasma to a Maxwellian, i.e. $n_M(\Phi) = n_0\exp(\Phi) = n_0(1 + \Phi + \ldots)$. However, for large potentials ($\Phi \gg 1$), the Maxwellian response n_M is significantly larger than the Dynamic response n_{tot}.

The method used to find the self-consistent potential Φ depends on the particular shielding problem. For example, the solution to the classic problem of the potential from a positively biased charged plane must be modified to take into account the collisionless "dynamic" density. The traditional Debye equation,

$$(\epsilon_0 kT/e)\nabla^2\Phi = en_0[\exp(\Phi) - 1], \quad (4)$$

valid for a collisionally-relaxed Maxwellian plasma, becomes

$$(\epsilon_0 kT/e)\nabla^2\Phi = en_0[\exp(\Phi)\operatorname{erfc}(\sqrt{\Phi}) + 2\sqrt{\Phi/\pi} - 1]. \quad (5)$$

The solutions to these two equations are shown in Fig. 2. Since both equations linearize to

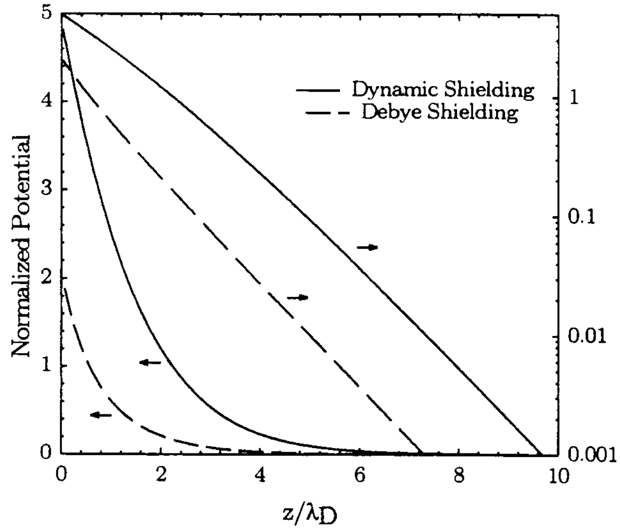

FIG. 2. Comparison of Debye and Dynamic shielding of a charged plane.

$$\nabla^2\Phi \approx \frac{e^2 n_0}{\epsilon_0 kT}\Phi = \frac{1}{\lambda_D^2}\Phi, \tag{6}$$

far from the charged plane both Debye and Dynamic shielded potentials decay exponentially with scale length λ_D. Near the plane the potential in the Dynamic case can be significantly greater than in the Debye case because $n_M(\Phi) \gtrsim n_{tot}(\Phi)$.

The collisionless response of the plasma to a negative well is substantially simpler than to a positive well because the negative well—effectively a potential hill—cannot trap electrons. In an infinite length plasma, the electron density decreases as[8,14]

$$n_-(\phi) = n_0 \exp(\Phi) \tag{7}$$

when $\Phi<0$. Since this matches the Boltzmann relation, the resulting shielding will be identical to the standard Debye response.

Since positive ions are not trapped by positive wells, ions with charge q will respond[8] to positive wells as $n_0 \exp(-q\Phi/e)$. Consequently, Dynamic-electron shielding will eventually be supplemented by Debye-like ion shielding when the ions are mobile.

Shielding in pure-electron plasmas is complicated by three-dimensional effects. Even though individual electrons are tied to magnetic field lines and behave one dimensionally, the response along different field lines differs, thereby suggesting a 3-D model. We have implemented such a 3-D model numerically, but have found that, for our particular experimental parameters, a simplified 1-D model is in good agreement with the 3-D numerical solutions and is sufficient to explain our data.[19]

Our 1-D model starts by assuming that all electrons, regardless of their radial position, respond to a common self-consistent well potential

$$\Phi = \Phi_w + \Delta\Phi_p(\Phi). \tag{8}$$

This assumption implies that the plasma radius is both small compared to the wall radius and on the same order as the Debye length, conditions met by our experiment. Here $V_w = kT\Phi_w/e$ is the externally-applied well potential, and $kT\Delta\Phi_p/e$ is the potential difference between the inside and the outside of the well that results from changes in the plasma density,

$$\Delta\Phi_p(\Phi) = \langle \phi(\Phi,r) - \phi(0,r)\rangle_r, \tag{9}$$

where $kT\phi(\Phi,r)/e$ is the potential of the plasma of density $n_{tot}(\Phi)$, $kT\phi(0,r)/e$ is the potential of a plasma of density $n_0(r)$, and $\langle\ \rangle_r$ denotes an average over the radius r, weighted by the plasma density. Since $\phi(\Phi,r)$ is a linear function of the density,

$$\Delta\Phi_p(\Phi) = C[n_{tot}(\Phi)/n_0 - 1], \tag{10}$$

where the density proportionality constant C is found by computing the weighted average. The complete response of the plasma to an external perturbation is found by self-consistently solving Eq. (8). To lowest order in Φ, the solution of Eq. (8) is

$$\Phi \approx \Phi_w/(1+|C|) \approx \Phi_w/(1+e|Vp|/kT). \tag{11}$$

The density inside the well is $n/n_0 \approx 1 + \Phi_w/(1+|C|) \approx 1+\Phi_w/(1+e|Vp|/kT)$. Here we have used $C \approx eV_p/kT \propto -r_p^2/\lambda_D^2$, where V_p is the central potential of the plasma cylinder and r_p is the plasma radius. The shielding efficacy, $1-\Phi/\Phi_w$, is plotted in Figs. 3 and 4. Note that perfect shielding corresponds to a shielding efficacy of one, and anti-shielding corresponds to negative shielding efficacies.

The collisionally-relaxed Debye response is found by replacing n_{tot} with n_M. For small applied potentials, the plasma again responds as $n/n_0 \approx 1+\Phi_w/(1+e|Vp|/kT)$. However, since $n_M(\Phi) \gtrsim n_{tot}(\Phi)$, the Debye response will once again be stronger than the Dynamic response.

This same formalism applies to the anti-shielding case. Here, the self-consistent potential reduces to

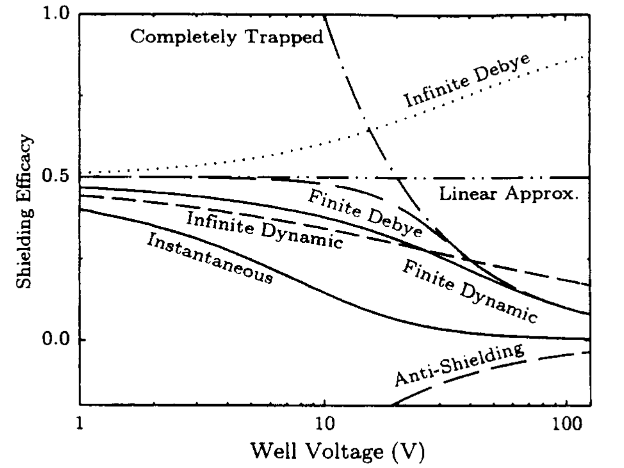

FIG. 3. Shielding efficacy $1-\Phi/\Phi_w$ vs. applied well voltage V_w at $T=5$ eV. For tutorial reasons, the density constant C is given the relatively small value $C=1$. Curves are shown for infinite length Debye and Dynamic theory, finite length ($L_{in}/L_{out}=1$) Debye and Dynamic theory, the linear approximation Eq. (11), and anti-shielding. Note that the efficacy decreases in finite length plasmas as the supply of free-electrons is exhausted at large V_w, and approaches the "Completely Trapped" in the well curve.

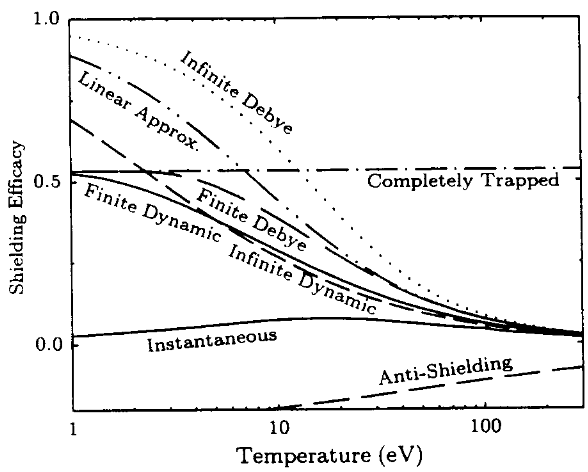

FIG. 4. Shielding efficacy $1-\Phi/\Phi_w$ vs. temperature T, for $V_w=30$ V and $C=8/T$. Curves are identified as in Fig. 3.

$$\Phi=\Phi_w+C[n_F(\Phi)/n_0-1]. \tag{12}$$

This equation has a remarkable solution: a low velocity phase space hole (cavity) which is self-sustaining, even when unanchored by any external fields. The existence of this solution is most readily seen in the limit $\Phi_w=0$, $V_p \ll kT$. There Eq. (12) reduces to $\Phi=2|C|(\Phi/\pi)^{1/2}$, which admits to two solutions: the trivial solution $\Phi=0$, and a self-sustaining cavity solution $\Phi=4C^2/\pi^2$. Such cavities have been observed experimentally[9,20] and in particle-in-cell (PIC) simulations.[21]

B. One-dimensional instantaneous theory

So long as the test well is created on a time-scale significantly greater than the well-transit time for a typical electron, the electron density will be given by Eq. (3) and shielding will result. However, slow test well creation is not necessarily required for shielding. Some shielding occurs even when the test well is created instantaneously. Immediately after such a test well is created, the electrons will not have had time to move and the density in the test well will be unchanged. However, as the untrapped electrons outside the test well redistribute, phase mixing will cause the density in the well to increase somewhat.[13] Since self-consistent phase mixing is a complicated, possibly turbulent process, general Instantaneous shielding can only be studied numerically. However, some analytic progress can be made in the restricted case of the shielding of an infinite-length plasma. After the electrons have phase mixed, the free-electron distribution function is given by $f(v)=f_0\{[v^2-2\Phi(z)]^{1/2}\}$, as before. If the applied potential is in the form of a square well, the density of trapped electrons is unchanged by any self-consistent adjustments, and, in the absence of any instabilities, the trapped distribution function then equals the initial electron distribution function, $f_0(v)$. As this "solution" has a possibly unstable cusp (see Fig. 1), it is not complete, and is only indicative of the true solution. Integrating gives the density:

$$n_I(\Phi)=n_0\{\exp(\Phi)+\mathrm{erf}(\sqrt{\Phi})[1-\exp(\Phi)]\}$$
$$=n_0(1+\Phi+\dots). \tag{13}$$

The degree of shielding is found by substituting this density into Eq. (6) or (10).

While Instantaneous shielding yields the same linear response as either Debye or Dynamic shielding, the nonlinear response is far weaker, as is reflected in the low shielding efficacies graphed in Figs. 3 and 4, where these shielding efficacies are calculated using approximate density given by Eq. (13). Shielding is normally thought to work best in the low temperature limit, and, indeed, the (infinite length) shielding efficacy for both Debye and Dynamic shielding goes to one in this limit. However, it is easy to show that the Instantaneous shielding efficacy scales as $|C|(T^3/\pi V_w^3)^{1/2}\rightarrow 0$ in this limit (assuming that C is held constant). Thus, Instantaneous shielding disappears in the very limit where Debye and Dynamic shielding work best. (Note that these conclusions have only been established for square-test wells in infinite-length plasmas, and ignore the effects of any instabilities due to the cusps in the distribution functions, and these effects may be important in many realistic situations.)

The results of this section assume that a steady state has been set up; i.e. that phase mixing has had time to occur ($t>1/\omega_p$) and that the trapped particles have made several bounces ($t>1/\omega_b$). Numerous authors[2,15–17] have analyzed the linear stage of Instantaneous shielding for small initial perturbations ($e\phi<kT$). As the method used by these authors apply only to the linear stage of the process, the results are only formally correct before trapped particles bounce, i.e. for times satisfying $1/\omega_p<t<1/\omega_b$.

C. Two and three-dimensional adiabatic theory

When electrons are no longer constrained to move in one dimension, their orbits will bend towards a positive test charge. If the test charge is two-dimensional, e.g. a line charge, the free-electron orbit bending near the test charge exactly compensates for the velocity increase there, and the net density of free electrons near the test line charge does not change. Consequently, previous authors[7,8] have claimed that test line charges are neither shielded nor anti-shielded by collisionless plasmas. Were this true, however, the line charge's logarithmic potential would eventually trap *all* the electrons and there would be *no* free electrons.

The exact 2-D response of the plasma is $n_{2D}=n_0[1+\Phi(r)]$, where $\Phi(r)$ is a radially symmetric, two-dimensional potential. The first term represents the density of the untrapped electrons, and is found by using the constancy of the total energy to solve Vlasov's equation. The second term represents the density of the trapped electrons, and is found using the fact that only electrons with zero initial velocity are trapped if the potential $\Phi(r)$ is turned on adiabatically. The exact self-consistent solution of the potential equation is $\Phi(r)=\Phi_0 K_0(r/\lambda_D)$, where K_0 is a modified Bessel function.[22] In the limit of large r, $K_0(r/\lambda_D)$ falls off exponentially with scale length λ_D.

In three dimensions, the electron orbits are sufficiently strongly bent towards a point test charge that the net electron density increases in the vicinity of the charge, despite the electrons' increased velocity. Thus, a point charge is collisionlessly shielded by the free electrons alone. The net electron density is

$$n_{3D}=n_0\left\{\exp(\Phi)\mathrm{erfc}(\sqrt{\Phi})+2\sqrt{\Phi/\pi}+\frac{4}{3\sqrt{\pi}}(\Phi)^{3/2}\right\}, \tag{14}$$

where the free-electron contribution is given by the first two terms,[7] and the trapped-electron contribution is given by the remaining term. While unimportant for very small values of Φ, the trapped electrons become the dominant shielding factor for $\Phi>2$.

D. Finite length, 1-D adiabatic theory

The above results assume that the background plasma is both Maxwellian and infinite in longitudinal extent. In many cases, however, including the pure-electron plasma experiments reported here, the plasma is of finite size. The initial Maxwellian distribution of such plasmas may be substantially modified by the creation of the test well. In these circumstances the plasma response is most readily calculated using the bounce adiabatic invariant, $J=\oint v_z dz$. When the test potential is square, the invariance of J implies

$$(L_{in}+L_{out})v_0=L_{in}v_{in}+L_{out}v_{out}, \tag{15}$$

where v_0 is the velocity of an electron before the test well is created, v_{in} (v_{out}) is the velocity of this same electron inside (outside) the test well after the test well is created, and L_{in} (L_{out}) is the plasma length inside (outside) the test well. Using energy conservation to relate the outside velocity to the inside velocity ($v_{out}^2=v_{in}^2+2\Phi$) and Eq. (15), we can construct a function which gives the initial velocity v_0 as a function of the final velocity inside or outside the well, e.g. $v_0=v_0(v_{in},\Phi)$. Since v_0 is a constant of the motion, the final distribution functions can be expressed in terms of the initial distribution function $f_0(v_0)$, for example, $f_{in}(v_{in})=f_0[v_0(v_{in},\Phi)]$.

As the test potential well deepens, the bounce invariant requires that electrons outside the well slowly lose energy. Some electrons will eventually reach $v_{out}=0$, indicating that these electrons are trapped. We can still use $f_T(v_{in})=f_0[v_0(v_{in},\Phi)]$ so long as we define v_0 by

$$(L_{in}+L_{out})v_0=L_{in}v_{in} \tag{16}$$

rather than Eq. (15).

Typical distribution functions for a positive test well are shown in Fig. 1. The densities inside and outside the well are found by integrating the appropriate distribution functions over v; the integral for the trapped component can be performed analytically,

$$n_T(\Phi)=n_0(1+L_{in}/L_{out})\mathrm{erf}\left(\frac{\sqrt{\Phi}}{1+L_{in}/L_{out}}\right), \tag{17}$$

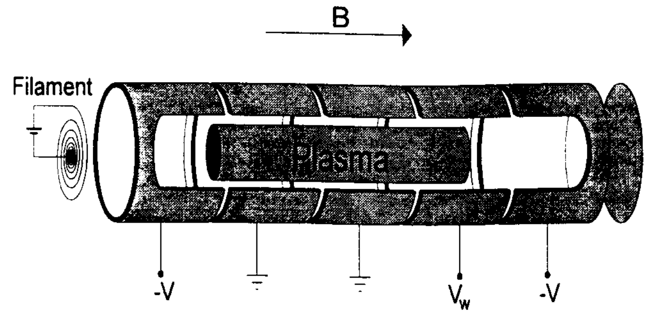

FIG. 5. Simplified experimental schematic.

but the integrals for the free electrons must be computed numerically. Once the total density $n_T(\Phi)$ is determined, the self-consistent solution to the finite-length shielding problem is readily obtained as outlined above.

Negative potential well can be similarly analyzed by swapping the values of L_{in} and L_{out}, inverting the sign of the applied potential, and calculating the total density in the new L_{out} region. The finite size case is substantially more complicated than the infinite case, and there is no closed form solution for the density like Eq. (7).

III. EXPERIMENT

The above theory was verified by pure-electron plasma experiments conducted in a Penning-Malmberg trap. Detailed descriptions of Penning-Malmberg traps can be found in the literature.[23,24] In our trap the plasma forms a cylinder aligned along the common axis of a series of eleven collimated, cylindrical, 1.905 cm radius electrodes (see Fig. 5). The electrodes are biased to create an electrostatic well, thereby providing axial confinement. A strong axial magnetic field (1800 G) provides radial confinement, and also ensures that the electron motion is one dimensional. The plasma is generated by thermionic emission from a hot tungsten filament. The plasma is heated by applying a broadband noise signal of variable amplitude to one of the confining cylinders, and the plasma temperature is measured using the standard dynamic-evaporation technique.[25] The plasma's radial profile is determined by radially scanning a pinhole across the plasma while the plasma is being dumped. The typical plasma radius is approximately 1 cm, the typical density ranges from $n_0\approx 1.2\times 10^7$ to 4×10^7 cm^{-3}, and the plasma temperature ranges from 1 to 20 eV.

We generate a test well by appropriately biasing the trap electrodes, and determine the plasma response by finding the total charge contained within the test well. This total charge equals the image charge on the test well electrode, and is measured by integrating the image current which flows onto the test well electrode. (To reduce undesirable coupling, we actually create the test well by leaving the well electrodes at ground and reverse biasing the remaining trap electrodes. The electrostatic well potential thus created is identical to that created by the more straightforward scheme of biasing the well electrodes themselves. The coupling is further minimized by guard electrodes.)

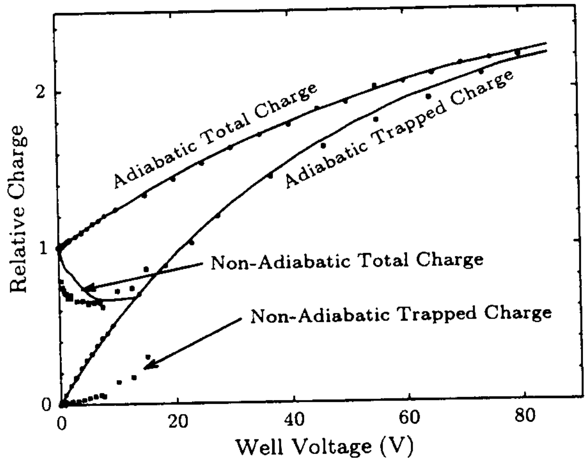

FIG. 6. Relative charge vs. well voltage. Relative charge is normalized to the charge found when $V_w=0$. Here $L_{in}/L_{out}=0.39$, $T=6.8$ eV and the density scale constant is $C=2.29$. Lines are from the analytic theory given in the text for the adiabatic case and from a PIC simulation in the non-adiabatic case.

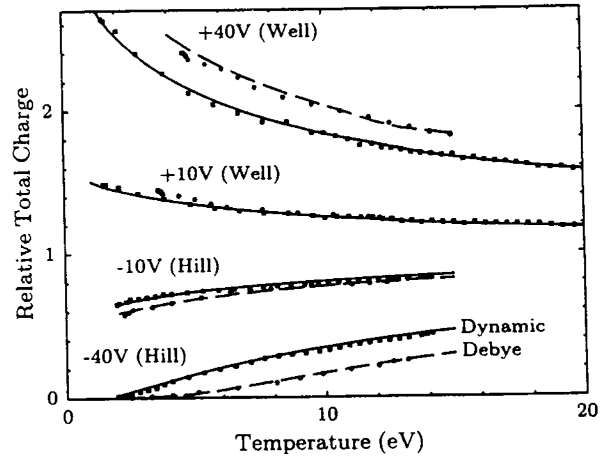

FIG. 7. Relative total charge vs. temperature. Here $L_{in}/L_{out}=0.36$ for wells and $L_{in}/L_{out}=0.80$ for hills. The measured plasma density varies with temperature such that the density constant C varies between $13.0/T$ to $16.3/T$ for wells, and $11.9/T$ to $15.4/T$ for hills. The lines are calculated from the theory described in the text using the value of C measured at each point.

Typically the test well is created adiabatically in a preexisting plasma. Figure 6 shows the resulting plasma response as a function of the well potential. As predicted by the theory, the charge in the test well increases as the test well depth is increased, thereby shielding the applied potential. The charge increase is due to charge trapped in the well. By allowing the free charge to escape and then measuring the remaining charge, we can approximately determine the amount of the trapped charge (see Fig. 6). Because the self-consistent condition changes to

$$\Phi=\Phi_w+C[n_T(\Phi)/n_0-1], \tag{18}$$

this trapped charge is not precisely the same as the trapped charge which exists before the free charge is allowed to escape. Consequently, the trapped charge shown in the figure does not follow the same scaling ($\sqrt{\Phi}$) as the initial trapped charge.

If the test well is created before the plasma is introduced into the well region (i.e. non-adiabatically), the sudden plasma expansion forms a phase-space cavity.[9,20,21] The resultant cavities are large and long-lived. As shown in Fig. 6, phase space holes with density modulations as large as approximately 50% are observed. The cavities appear to oscillate from one end of the trap to the other, last for several milliseconds (thousands of plasma periods and axial bounces), and increase in size for well potentials less than ten volts. Instabilities set in at about 10 V, and prevent us from measuring the response for well voltages greater than 20 V. The data is in rough agreement with a PIC simulation.[26]

Figure 7 shows the total well charge vs. the plasma temperature for several well potentials. The positive potentials form potential wells for the electrons and are shielded by resultant excess of electrons. The negative potentials form potential hills for the electrons and are shielded by the resultant deficit of electrons. The square points in the figure show the Dynamic response; the measurements are taken in a time much less than the collision time. The round points show the Debye response; the well is created over a length of time of ~ 1 s, far longer than the collision time for the plasma. As predicted by the theory, the Debye response is stronger than the Dynamic response for both hills and wells.

Figure 8 shows the trapped charge remaining after the free particles have escaped as a function of the plasma temperature. Similar to Fig. 6, the change in the self-consistent equation makes this charge somewhat different than the trapped charge before the free particles escape. As expected, more charge is trapped in the Debye case than in the Dynamic case.

Finally, Fig. 9 contrasts the distribution function found inside a ten volt well with the initial, presumably Maxwellian distribution function found before the application of the well. The distribution functions were found by first using an electrode to split the plasma into inside and outside (the

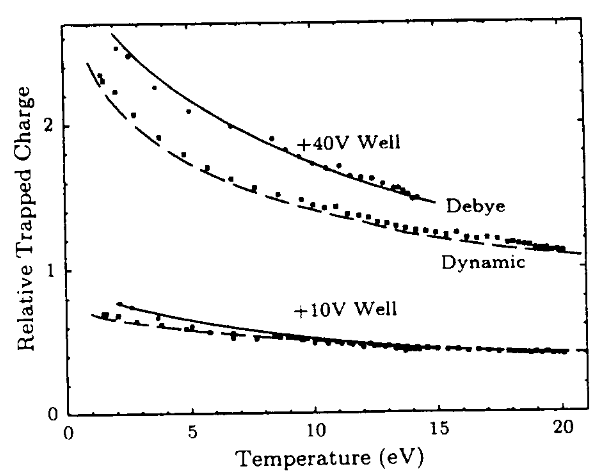

FIG. 8. Relative trapped charge vs. temperature. Parameters are identical to well parameters in Fig. 7.

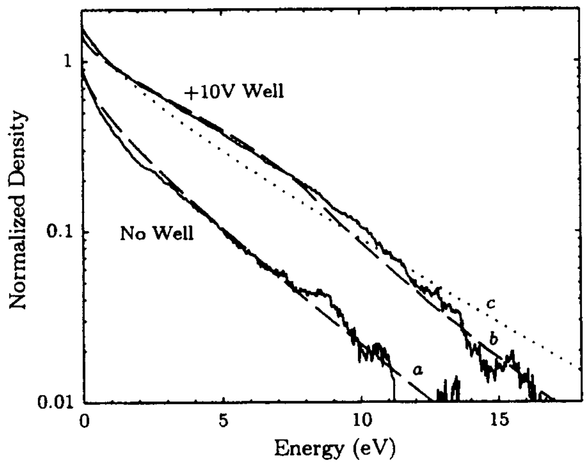

FIG. 9. Experimental distribution functions. Shown are the experimental data (solid lines) before and after the application of a + 10 V well, the (a) best fit Maxwellian ($T=3.8$ eV) to the initial plasma, the (b) calculated "Dynamic" distribution in the well, and (c) the calculated Maxwellian ($T=5.2$eV) to which the experimental data should relax due to collisions. Here $L_{in}/L_{out}=0.77$.

well) regions, and then measuring the number of electrons that escape as a function of the end confinement-barrier energy E. We avoid the confounding effects of the plasma space charge by using a very tenuous plasma ($C=0.26$). This is a difficult measurement, and noise limits us to presenting only integrated data, $n(E)=2n_0\int_{\sqrt{2eE/kT}}^{\infty}f(v)dv$. Nonetheless, the distribution function inside the well is clearly non-Maxwellian.

IV. DISCUSSION

We have shown that 1-D collisionless shielding relies on trapping, and without trapping, a plasma anti-shields a positive test charge. Since the electrons in highly magnetized plasmas respond one dimensionally, 1-D shielding is common. Further, the standard 2-D and 3-D derivations require modification to model trapping correctly, and the plasma response in 2-D is qualitatively different than reported elsewhere. The distribution functions found in collisionless (Dynamic) shielding are non-Maxwellian, and the Dynamic response can be far weaker than the equilibrium Debye response in the nonlinear regime. We have experimentally demonstrated both shielding and anti-shielding, and our results are in excellent agreement with the predictions of 1-D Dynamic and Debye theory.

ACKNOWLEDGMENTS

We thank B. R. Beck, C. K. Birdsall, K. Cartwright, D. Cooperberg, D. Durkin, R. Gould, T. M. O'Neil, and J. S. Wurtele for their helpful comments.

This work was supported by the Office of Naval Research.

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