Marty B. Halpern Memorial:
From mesons to orbifolds via affine Lie algebras

March 29-30, 2019, UC Berkeley

Infinite dimensional Lie algebras:
from Marty and Korkut to brane creation

Ori Ganor (UC Berkeley)

My first correspondence with Marty

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Don't have original email, unfortunately, but here's how we modified our paper as a result . . .

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“Historically, our procedure was pioneered by Halpern [12, 13, 14]. In particular, our starting point is Halpern’s 1977 field strength formulation[12] of Yang-Mills. . . .”

[OJG & Sonnenschein, 1995]

[12] **M.B. Halpern,** Phys. Rev. **D16**(1977) 1798

[13] **M.B. Halpern,** Phys. Rev. **D16**(1977) 3515

[14] **M.B. Halpern,** Nucl. Phys. **B139**(1978) 477,
Phys. Rev. **D19**(1979) 517

[15] **M.B. Halpern,** Phys. Rev. **D19**(1979) 517

Affine Lie Algebras

"...two physicists, **Bardakci and Halpern,** in ref. 4 constructed a representation of the subalgebra $\widetilde{\mathfrak{gl}}(l)$ of $\widetilde{\mathfrak{o}}(l)$ in the space $V((2Z+1)^l)$ (see formulas 3.1-3.11 in ref. 4). **At that time the theory of affine Lie algebras began to take its first steps.**"

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ref. 4 –
Korkut Bardakci and Martin B. Halpern,
New dual quark models, Phys. Rev. D **3**, 2493 (1971).

Marty's work on orbifolds

(See also Jan's talk!)

$\hat{A}^{(r)}(m + \frac{r+\epsilon}{\lambda}) \equiv \lambda^{1-\Delta} A_\Delta(\lambda m + r + \epsilon)$ Orbifold induction procedure

(Borisov, **Halpern**, Schweigert, 1997)

$\hat{c} = 26K, \qquad K = 2, 3, 4, \ldots$ Permutation type orbifolds

$\frac{U(1)^{26K}}{H(\text{perm})_K}, \qquad \frac{U(1)^{26K}}{\mathbb{Z}_2(\text{w.s.})}$ (**Halpern**, 2007)

Twisted sector Virasoro generators

$$\hat{L}_{\hat{j}j}(m + \frac{\hat{j}}{f_j(\sigma)}) = \frac{13}{12}\delta_{m+\frac{\hat{j}}{f_j(\sigma)},0}\left(f_j(\sigma) - \frac{1}{f_j(\sigma)}\right)$$

$$-\frac{1}{2f_j(\sigma)}\eta^{ab}\sum_{\hat{\ell}}^{f_j(\sigma)-1}\sum_{p\in\mathbb{Z}} : J_{0a\hat{\ell}j}(p + \frac{\hat{\ell}}{f_j(\sigma)}) J_{0b,\hat{j}-\hat{\ell},j}(m - p + \frac{\hat{j}-\hat{\ell}}{f_j(\sigma)}) :_M$$

$$\sum f_j(\sigma) = K$$

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- The Orbifolds of Permutation-Type as Physical String Systems at Multiples of $c = 26$. I. Extended Actions and New Twisted World-Sheet Gravities, [arXiv:hep-th/0703044]

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- The Orbifolds of Permutation-Type as Physical String Systems at Multiples of $c = 26$ V. Cyclic Permutation Orbifolds [arXiv:0705.2062]

And more . . .

- The Orbifold-String Theories of Permutation-Type: I. One Twisted BRST per Cycle per Sector, [arXiv:1008.1453]
- The orbifold-string theories of permutation-type: II. Cycle dynamics and target space-time dimensions, [arXiv:1008.2576]
- The Orbifold-String Theories of Permutation-Type: III. Lorentzian and Euclidean Space-Times in a Large Example, [arXiv:1009.0809]
- The Lorentzian Space-Times of the Orientation-Orbifold String Systems, [arXiv:1010.1893]

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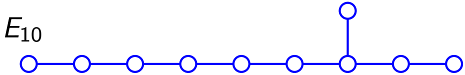
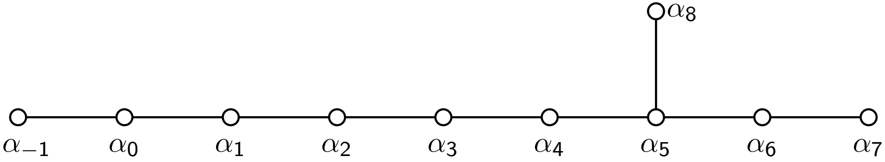


E_{10}


α_8
α_{-1} α_0 α_1 α_2 α_3 α_4 α_5 α_6 α_7

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- Does E_{10} know about branes?

Branes, Fluxes, Roots

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$$E_{10} \text{ root:} \quad \boxed{\alpha = (n_1, n_2, \ldots, n_{10})}$$

$$n_i \in \mathbb{Z}, \qquad \sum_1^{10} n_i \in 3\mathbb{Z}, \qquad 2 \geq \alpha^2 \equiv \sum_1^{10} n_i^2 - \frac{1}{9}\Big(\sum_1^{10} n_i\Big)^2.$$

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Flux

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Flux created by instanton with action $2\pi I_\alpha \rightarrow$

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Flux created by instanton with action $2\pi I_\alpha \to$ root

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Flux C_{123}	created by instanton	with action $2\pi I_\alpha \rightarrow$	root

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g_{12}/g_{22}	KK	$2\pi R_1 / R_2$	$(1, -1, 0, \ldots, 0)$
$\widetilde{C}_{123456}$	M5	$2\pi R_1 \cdots R_6$	$\ldots$

Branes, Fluxes, Roots

$$E_{10}\text{ root:}\quad \boxed{\alpha = (n_1, n_2, \ldots, n_{10})}$$

$$n_i \in \mathbb{Z}, \qquad \sum_1^{10} n_i \in 3\mathbb{Z}, \qquad 2 \geq \alpha^2 \equiv \sum_1^{10} n_i^2 - \frac{1}{9}\Big(\sum_1^{10} n_i\Big)^2.$$

$$ds^2 = -dt^2 + \sum_1^{10} R_i^2 dx_i^2, \qquad \boxed{I_\alpha \equiv R_1^{n_1} R_2^{n_2} \cdots R_{10}^{n_{10}}} \qquad (M_{\rm pl} = 1)$$

Flux	created by instanton	with action $2\pi I_\alpha \to$	root
C_{123}	M2-brane	$2\pi R_1 R_2 R_3$	$(1,1,1,0,\ldots,0)$
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Roots and Chern-Simons terms

$\boxed{\alpha + \beta = \gamma} \rightarrow$ CS term

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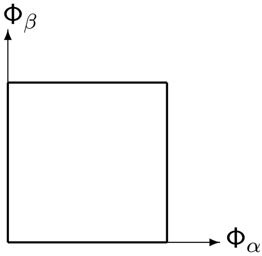

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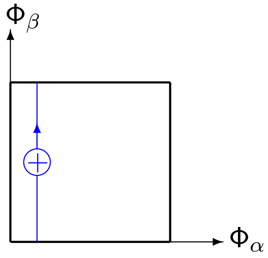

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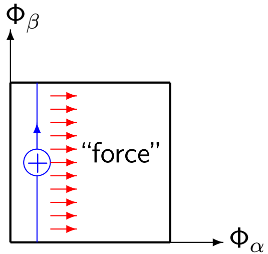

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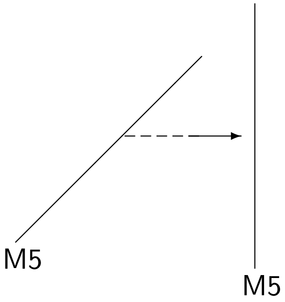
$$\boxed{\alpha+\beta=\gamma} \Longrightarrow \begin{array}{ccc} (\Phi_\gamma) & & \\ S^1 & \longrightarrow & X \\ & & \downarrow \\ & & T^2 \\ & & (\Phi_\alpha, \Phi_\beta) \end{array} \quad \subset \text{Nilp}(\mathbb{Z})\backslash E_{10}(\mathbb{R})/KE_{10}$$

Brane creation



(Hanany, Witten, 1996)

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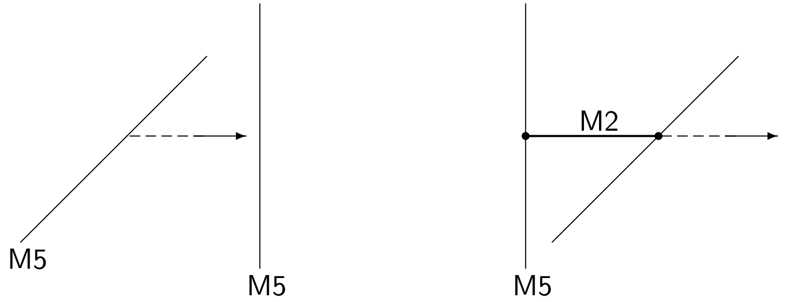

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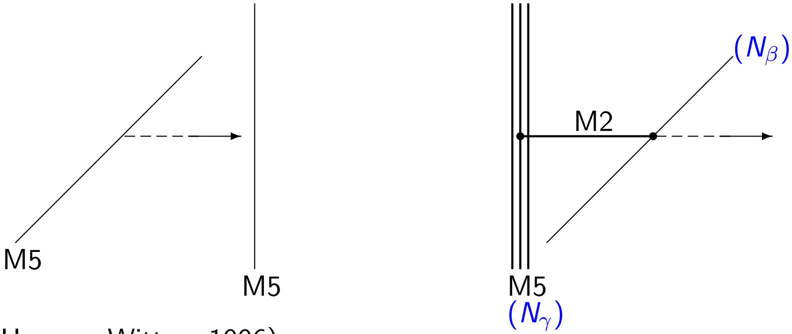

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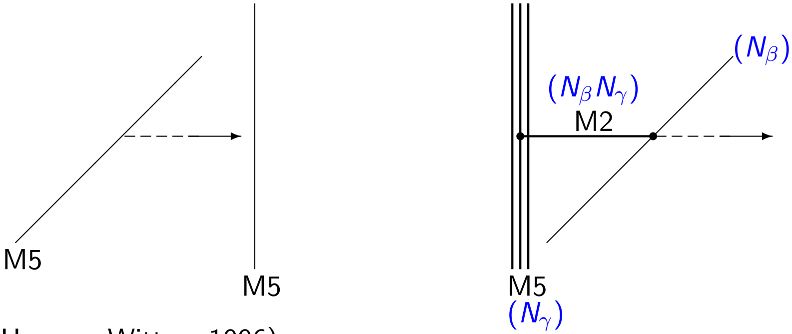

(Hanany, Witten, 1996)

Simple roots - SKIP

Root	Field
$\alpha_{-1} = (1, -1, 0, 0,\ 0, 0, 0, 0,\ 0, 0)$	g_{12}/g_{22}
$\alpha_0 = (0, 1, -1, 0,\ 0, 0, 0, 0,\ 0, 0)$	g_{23}/g_{33}
$\alpha_1 = (0, 0, 1, -1,\ 0, 0, 0, 0,\ 0, 0)$	g_{34}/g_{44}
$\alpha_2 = (0, 0, 0, 1,\ -1, 0, 0, 0,\ 0, 0)$	g_{45}/g_{55}
$\alpha_3 = (0, 0, 0, 0,\ 1, -1, 0, 0,\ 0, 0)$	g_{56}/g_{66}
$\alpha_4 = (0, 0, 0, 0,\ 0, 1, -1, 0,\ 0, 0)$	g_{67}/g_{77}
$\alpha_5 = (0, 0, 0, 0,\ 0, 0, 1, -1,\ 0, 0)$	g_{78}/g_{88}
$\alpha_6 = (0, 0, 0, 0,\ 0, 0, 0, 1,\ -1, 0)$	g_{89}/g_{99}
$\alpha_7 = (0, 0, 0, 0,\ 0, 0, 0, 0,\ 1, -1)$	$g_{9\natural}/g_{\natural\natural}$
$\alpha_8 = (0, 0, 0, 0,\ 0, 0, 0, 1,\ 1, 1)$	$C_{89\natural}$

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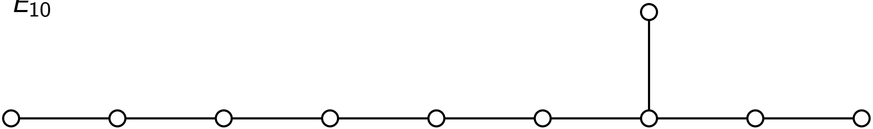
- Setup is a U-dual of Hořava-Witten's orbifold ($S^1/\mathbb{Z}_2$)

$E_{10}/\mathbb{Z}_2$

E_{10}



Define $\mathbb{Z}_2$ action on root lattice.

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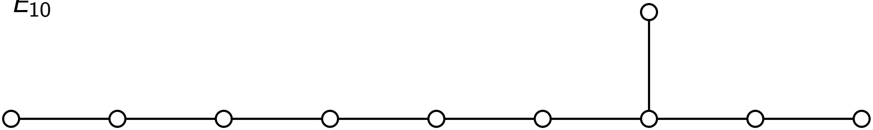

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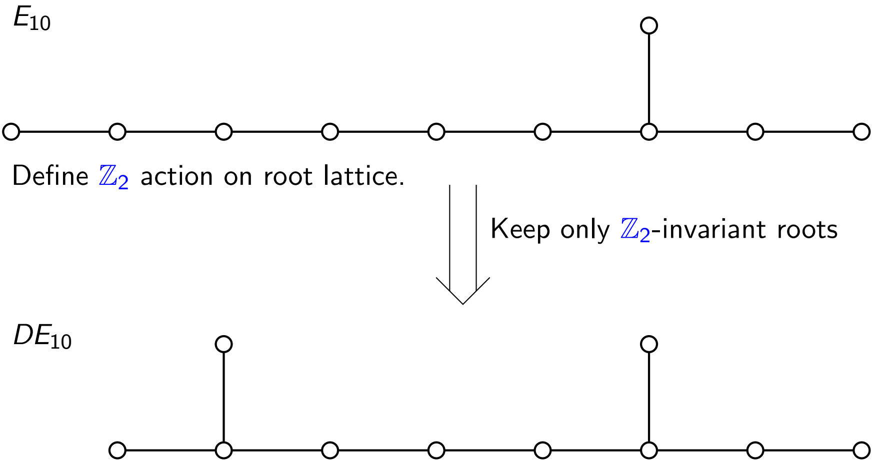

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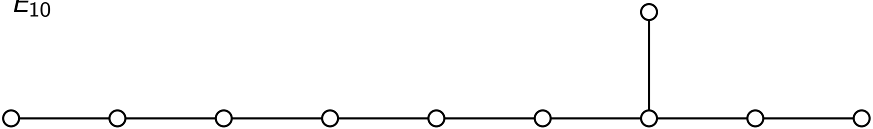

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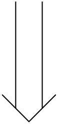

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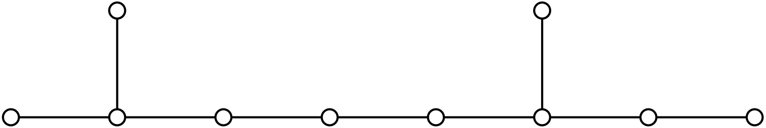
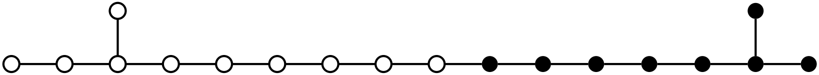
DE_{10}



[Brown, Ganguli, OJG, Helfgott, 2005]

M-theory on $T^{10}/\mathbb{Z}_2$ with 16 M5's

$DE_{18(10)}$ incorporates 16 M5-branes



The Dynkin diagram of the real form $DE_{18(10)}$ of DE_{18}.

White circles correspond to noncompact directions.

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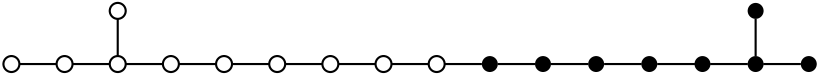

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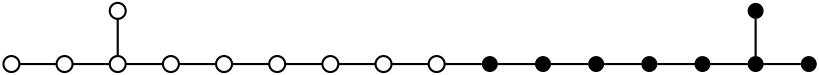

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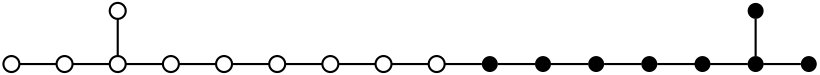

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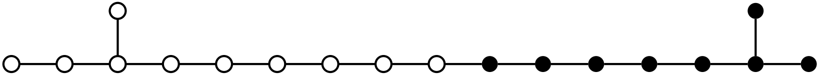

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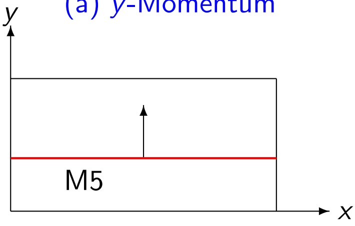
Examples of new fluxes

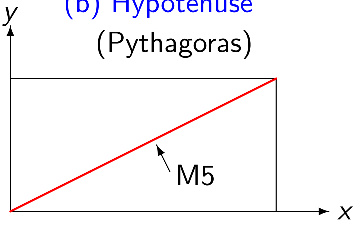
x, y are two periodic directions.

M5-brane is wrapped on x but not on y.

(a) y-Momentum

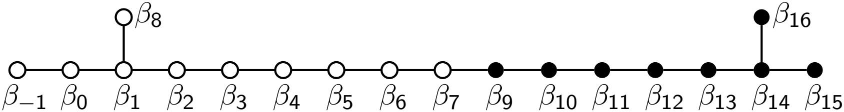


(b) Hypotenuse
(Pythagoras)



Note: Relation between $DE_{18(10)}$ and DE_{10}

$DE_{18(10)}$



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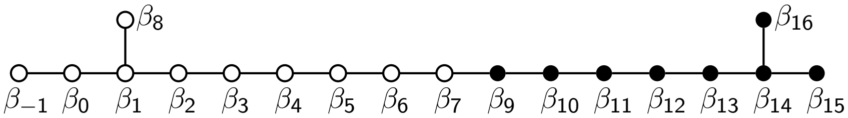
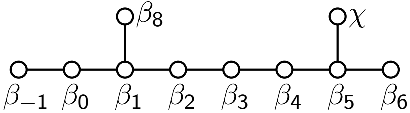


Commutant of D_8 (black nodes):

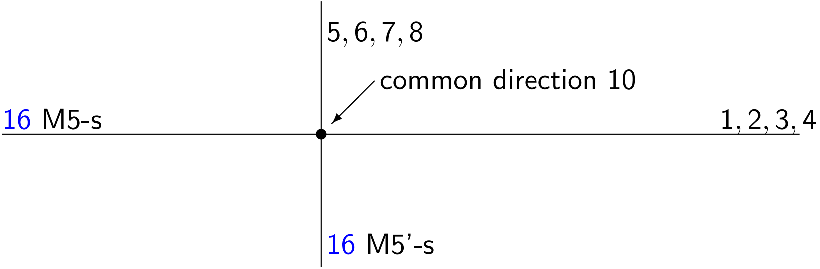
$\mathfrak{g}^{(com)} \simeq DE_{10} \subset E_{10}$



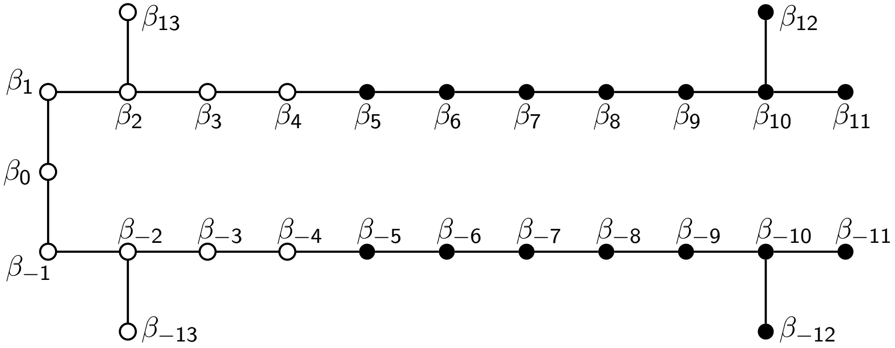
$$\chi = (\beta_6 + 2\beta_7) + (2\beta_9 + 2\beta_{10} + 2\beta_{11} + 2\beta_{12} + 2\beta_{13} + 2\beta_{14} + \beta_{15} + \beta_{16}).$$

The double orbifold: M on $T^{10}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

Direction	1	2	3	4	5	6	7	8	9	10
$\mathbb{Z}_2$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$	$+$
$\mathbb{Z}_2'$	$-$	$-$	$-$	$-$	$+$	$+$	$+$	$+$	$-$	$+$
M5	$=$	$=$	$=$	$=$						$=$
M5'					$=$	$=$	$=$	$=$		$=$
24 M2's									$=$	$=$



Dynkin diagram for the double orbifold



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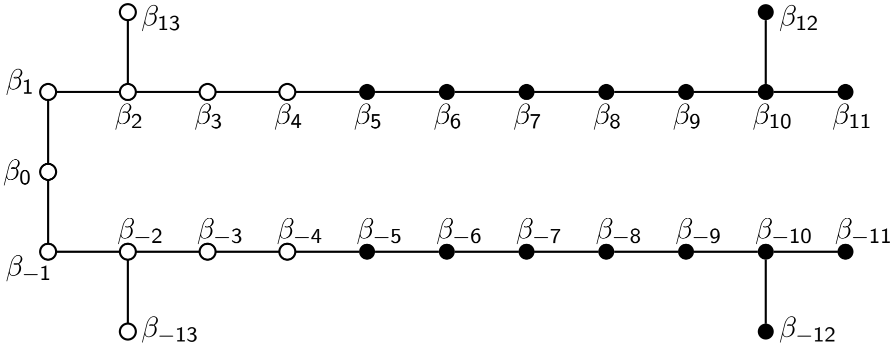

TT_{27} – Rank-27 Infinite Dimensional Lie Algebra

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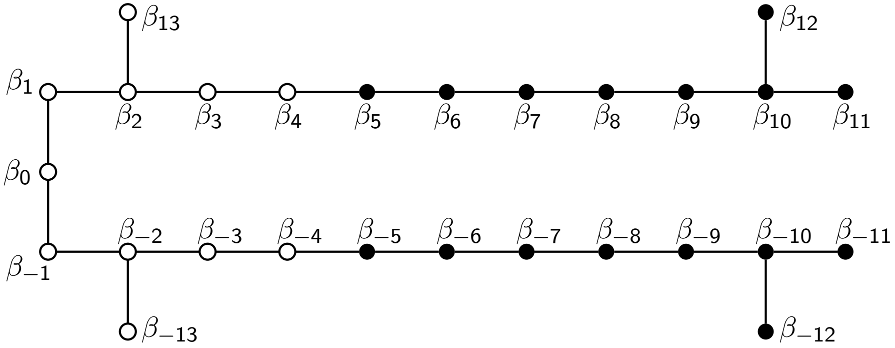
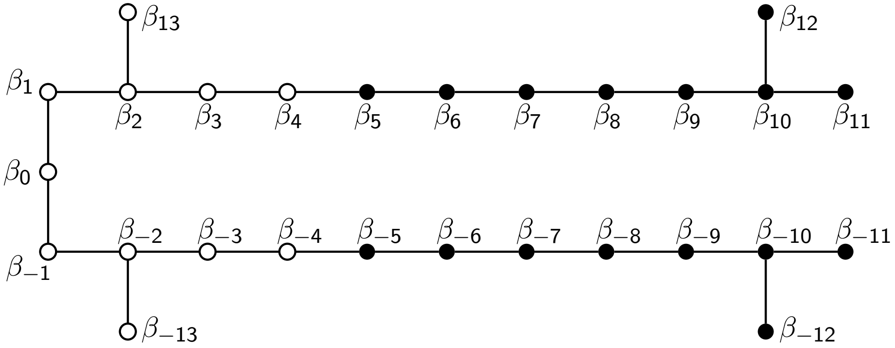


TT_{27} – Rank-27 Infinite Dimensional Lie Algebra

Captures fluxes of M-theory $T^{10}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

Dynkin diagram for the double orbifold

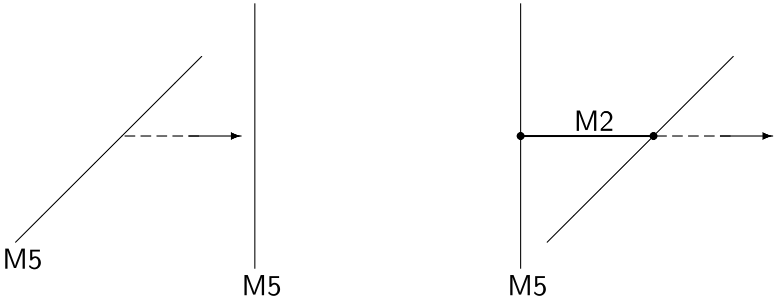


TT_{27} – Rank-27 Infinite Dimensional Lie Algebra

Captures fluxes of M-theory $T^{10}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$

And more?!?!

Why are created-branes associated with flux?



(Hanany, Witten, 1996)

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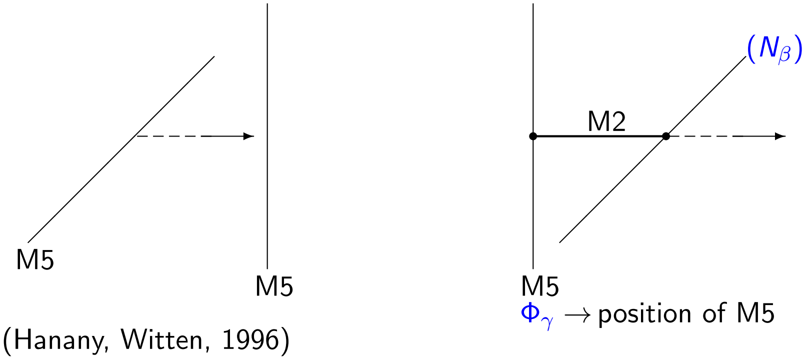

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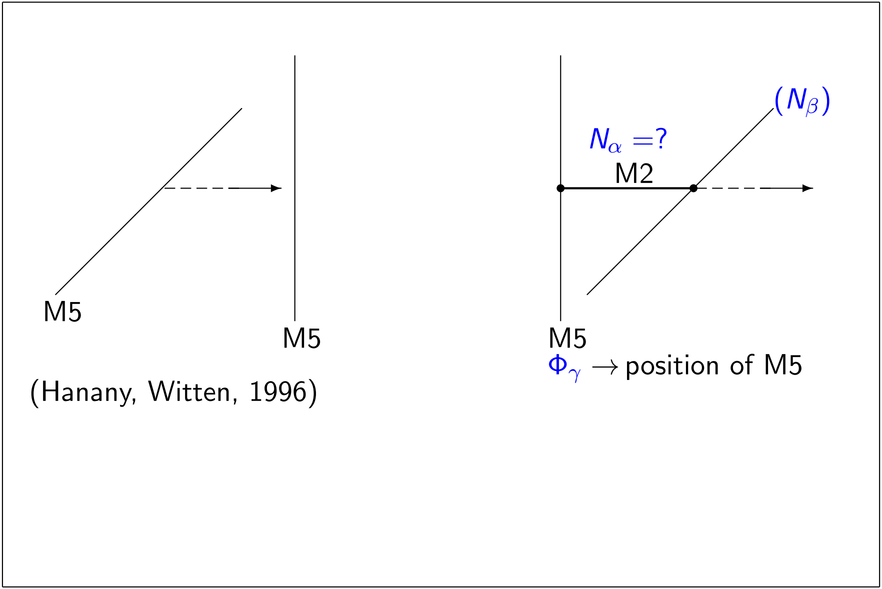

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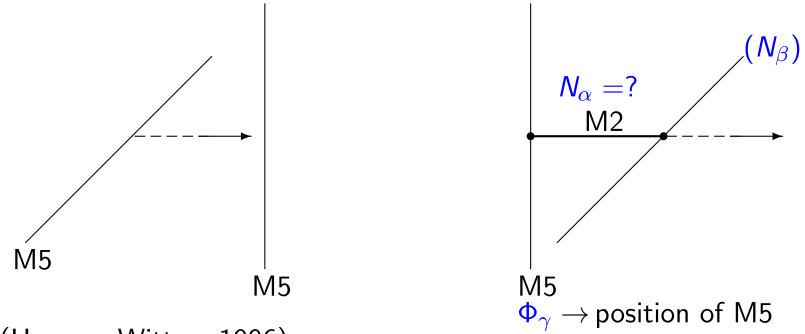

(Hanany, Witten, 1996)

Let's go back to Korkut and Marty's 1970 paper!

Back to Bardakci-Halpern

$$S_0 = \sum_{n=0}^{\infty}(n+\tfrac{1}{2})[b^\dagger(n)b(n) + d^\dagger(n)d(n)] \qquad \text{Equation (3.4')}$$

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Filled up to level N_α:

Energy $\propto \frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \cdots + (N_\alpha - \frac{1}{2}) = \frac{1}{2} N_\alpha^2$

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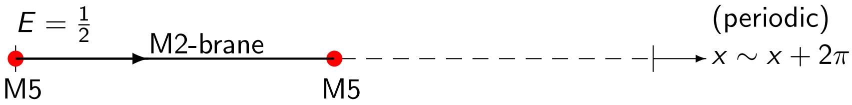
(1+1d bosonization.)

flux associated with M2-branes



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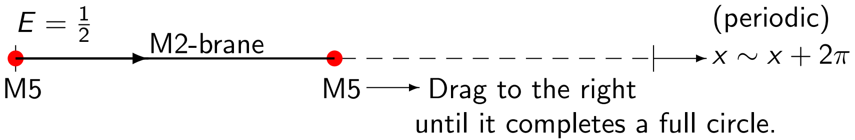

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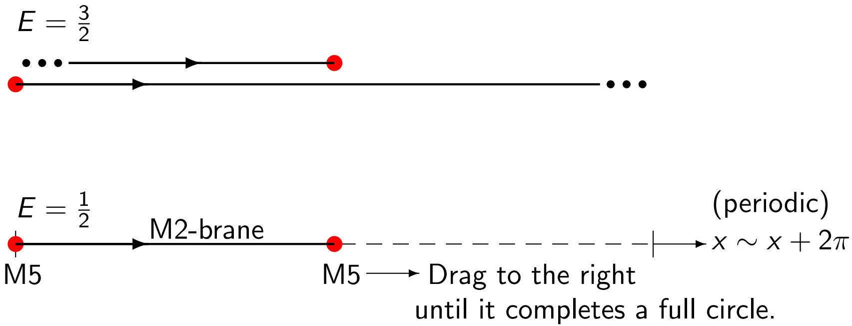

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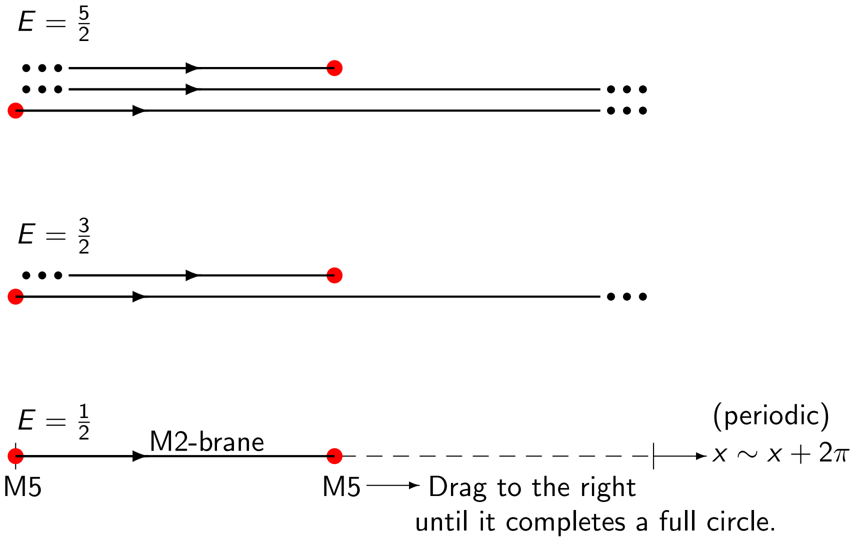
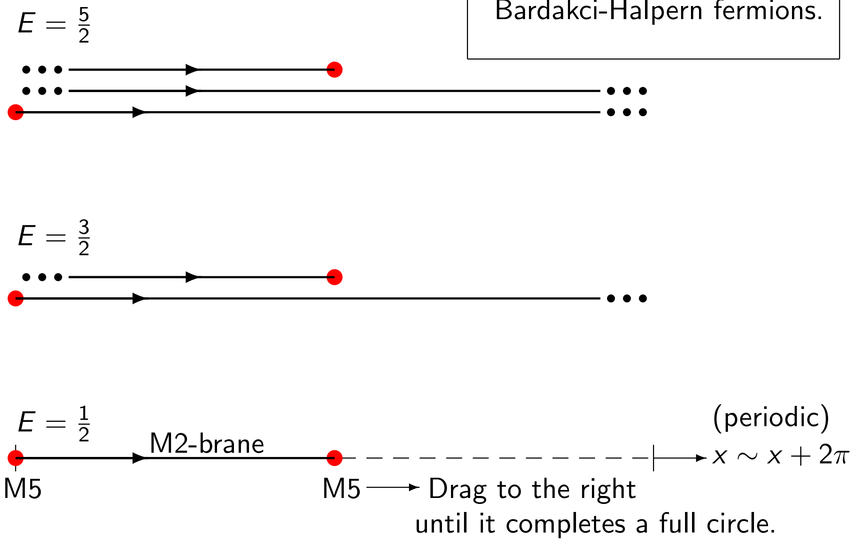
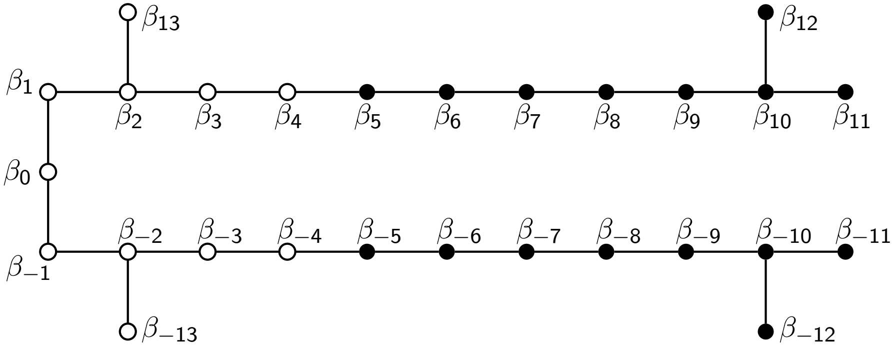


flux associated with M2-branes

This is T-dual to Bardakci-Halpern fermions.



The root $\to$ flux of M2-branes



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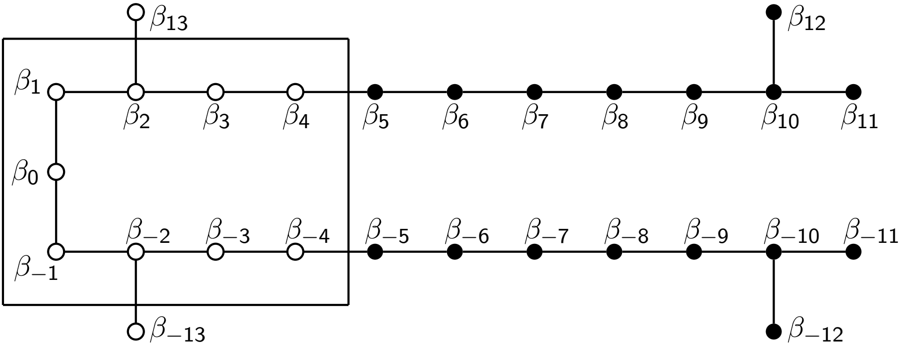

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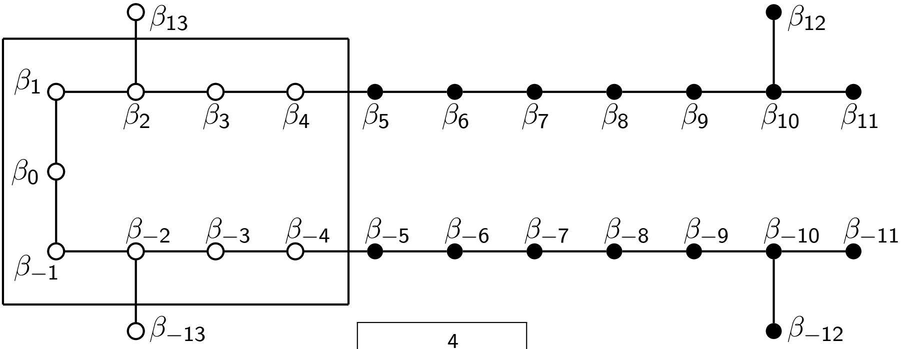
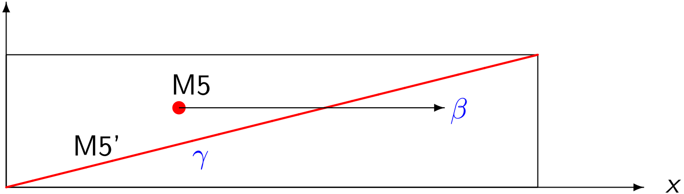


$$\alpha = \sum_{k=-4}^{4} \beta_k$$

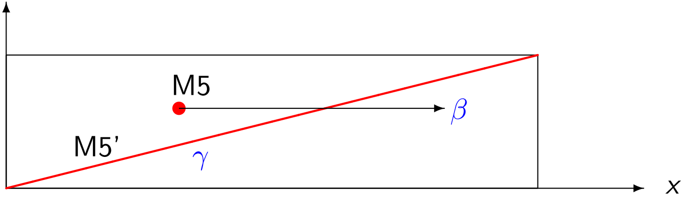
SKIP

$$\zeta := \sum_{k=-4}^{4} \beta_k \quad = \quad (\tfrac{\mathbf{1}}{\mathbf{2}}, \tfrac{\mathbf{1}}{\mathbf{2}}, \tfrac{\mathbf{1}}{\mathbf{2}}, \tfrac{\mathbf{1}}{\mathbf{2}},\ \tfrac{\mathbf{1}}{\mathbf{2}}, \tfrac{\mathbf{1}}{\mathbf{2}}, \tfrac{\mathbf{1}}{\mathbf{2}}, \tfrac{\mathbf{1}}{\mathbf{2}},\ \mathbf{1}, \mathbf{1};\ 0;$$

$$[-1, 0, 0, 0, 0, 0, 0, 0];\ [-1, 0, 0, 0, 0, 0, 0, 0]),$$



M2-brane creation as an M5-brane is dragged through M5'.



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$$\beta = \beta_{-12} + \beta_{-11} + 2\sum_{k=5}^{10} \beta_{-k} + \sum_{k=1}^{4} \beta_{-k} \to \text{M5-Momentum}$$

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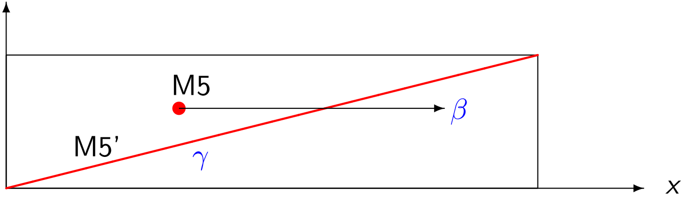

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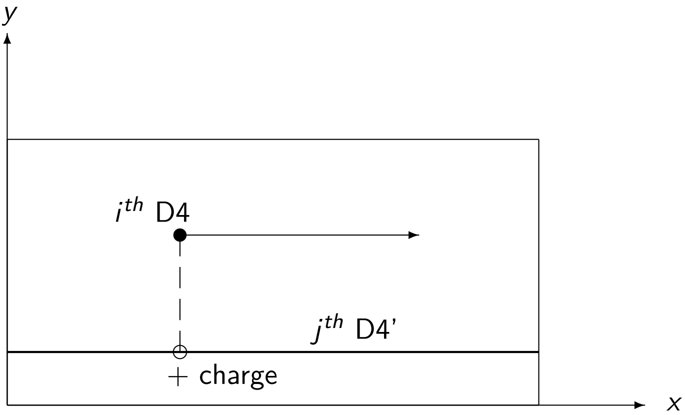
$$\boxed{\gamma = \alpha + \beta}$$

Another test



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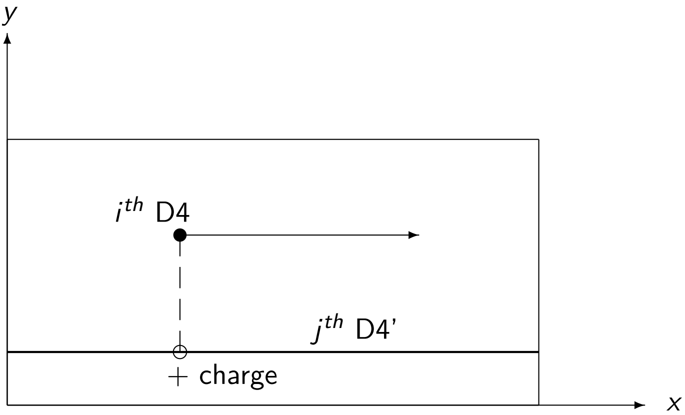

Dragged string-endpoint creates electric flux.

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- Marty's discoveries introduced us to ∞-dimensional Lie algebras (among many other things);
- Almost 50 years later, we are still exploring new applications;
- There seems to be a connection to branes in M-theory;
- (Also dual to M-theory on $K_3 \times K_3$ and M-theory on CY; created-branes are dual to G-flux.)

Outlook and open questions

- M2-brane interactions from automorphic forms on TT_{27}?
  (See Obers-Pioline, Kleinschmidt-Nicolai-Palmqvist, . . .)

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Outlook and open questions

- M2-brane interactions from automorphic forms on TT_{27}? (See Obers-Pioline, Kleinschmidt-Nicolai-Palmqvist, ...)
- Relations to dynamics of exotic branes? [de Boer, Shigemori, 2010]
- Can autmorphic forms on E_{10} and other infinite dimensional Lie groups provide new predictions for M-theory dynamics?

Forever grateful to Marty,

for his friendship, mentorship and discoveries.

More of Marty's results

$$D = \int d^4x \left[-\frac{1}{2g} G \mathfrak{F}(\mathcal{J}(G)) - \frac{1}{4} G^2 - \chi \frac{g}{2} \mathcal{G} \chi \right]$$

Field-strength formulation of quantum chromodynamics [Halpern, 1977]

$$\langle \mathrm{Tr}_A U[C] \rangle \sim N^2 (e^{-2\sigma_F A[C]} + N^{-2} e^{-4\sigma_F P[C]})$$

Suppression of color screening at large N [Greensite and Halpern, 1982]

$$H_5 = \int d^D x \left[-\frac{1}{2} \frac{\delta^2}{\delta\phi^2} + \frac{1}{8\hbar^2} \left(\frac{\delta S}{\delta \phi} \right)^2 - \frac{1}{4\hbar} \frac{\delta^2 S}{\delta \phi^2} \right] = \frac{1}{2} \int d^D x R^+(x) R(x) \geq 0.$$

Stabilizing bottomless action theories [Greensite & Halpern, 1984]

$$|\nu = \tfrac{1}{3}\rangle = r^2 \left\{ -K_{2/3}\left(\tfrac{2g}{3} r^3\right) + e^{-i\phi} K_{1/3}\left(\tfrac{2g}{3} r^3\right) \overline{\psi}_1 \overline{\psi}_2 \right\} |0\rangle, \qquad z = re^{i\phi}$$

Supersymmetric Ground State Wave Functions [Claudson and Halpern, 1985]

and much more . . .