15. Atoms which are initially in an unpolarized ground state with $\mathrm{J}=3 / 2$ are subject to optical pumping with light which is near-resonant with a transition to an excited state with $J^{\prime}=1 / 2$. Assuming that all atoms excited to the $J^{\prime}=1 / 2$ state decay to a 'trap' state other than the ground state and that other relaxation processes can be neglected, find the $4 \times 4$ density matrix describing the Zeeman sublevels of the $\mathrm{J}=3 / 2$ after the optical pumping is complete. Let the quantization axis be the $\mathbf{z}$-axis of a coordinate system. Consider the following two cases of light polarization:
a) right circular polarization and
b) linear polarization along $\mathbf{x}$.
16. An ensemble of atoms is excited from the ground state $G$ to a state $H$ with laser light, which is near resonant to the $\mathrm{G} \rightarrow \mathrm{H}$ transition with residual detuning $\delta$. The state H spontaneously decays back to the ground state and to a metastable state $\mathrm{G}^{\prime}$ (see figure). Describe the spectrum of the fluorescence in the two limiting cases:
a) The laser light is monochromatic.
b) The laser light has bandwidth
$\Delta \nu_{\mathrm{L}} \gg \gamma / 2 \pi ; \Delta v_{\mathrm{L}} \gg \delta$, where $\gamma / 2 \pi$ is the natural width of the state $H$ in frequency units. Neglect all sources of broadening rather than the radiative
 decay of the state H .
c) Suppose two-level atoms (levels $\mathrm{H}, \mathrm{G}$ ) are moving onto a monochromatic light beam. What is the frequency of the fluorescent photons observed in the orthogonal direction?
17. Suppose you have a beam of linearly polarized light of intensity $I$. You need to rotate the polarization plane by $\pi / 2$ (e.g. change the polarization from vertical to horizontal). It so happens that the only optical components you have at your disposal are highquality dichroic polarizers (elements that transmit one linear polarization and totally absorb the orthogonal polarization). The good news is that you have a whole box of
them. How do you rotate the polarization? What is the maximal achievable intensity of the output light with the desired polarization?
18. Consider the linear Macaluso-Corbino effect (Faraday rotation near a resonance absorption line) for a $\mathrm{J}=0 \rightarrow \mathrm{~J}=1$ transition. Assume that the only line-broadening mechanism is the natural with $\gamma_{0}$ of the upper state. What is the spectral dependence of the rotation? How does the peak rotation scale with $\gamma_{0}$ ? (Compare this to how the peak absorption scales with $\gamma_{0}$.) What is the magnetic field dependence of the rotation on resonance? What is the maximum rotation angle per one absorption length?
