Shielding from instantaneously and adiabatically applied potential wells in collisionless plasmas

A. B. Reimann and J. Fajans
Department of Physics, University of California at Berkeley, Berkeley, California 94720-7300

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The response of collisionless plasmas to an applied potential can differ from classic Debye shielding, and when the potential is rapidly applied ("instantaneous shielding"), the shielding can differ from when it is slowly applied ("adiabatic shielding"). Experiments demonstrate that when a test potential well is applied to a one-dimensional pure-electron plasma, instantaneous and adiabatic shielding are similar for small potentials, but that instantaneous shielding is weaker than adiabatic shielding for large potentials. These results have been confirmed with particle-in-cell computer simulations. Simulations also show that the peculiar distribution functions of instantaneously and adiabatically shielded plasmas agree with theoretical predictions. [DOI: 10.1063/1.1455633]

I. INTRODUCTION

The shielding of a test charge by the accumulation of oppositely charged particles in the vicinity of the test charge is one of the most fundamental properties of a plasma. However, although the exact method and time scale on which the test charge is introduced into the plasma can have important consequences, these factors are generally ignored. Shielding was first studied in 1923 by Debye and Hückel[1] for ionic solutions, and their theory is easily extended to plasmas in thermal equilibrium; plasma "Debye shielding" assumes that the time scales are long enough for collisions to rethermalize the plasma in the vicinity of the test charge. Most elementary discussions of shielding implicitly or explicitly assume such collisions. Yet it is well known that shielding in plasmas does not actually require collisions. More sophisticated treatments do not require collisions, but implicitly assume that the test charge is placed into the plasma instantaneously. We will call this regime "instantaneous shielding." More recent work examined a third regime where the test charge is placed into the plasma or turned on slowly. This regime is called "adiabatic shielding." [2–5] Finally, if the plasma is allowed to flow into the vicinity of a previously existing test charge, rather than the more normal circumstance in which the test charge is placed in an previously existing plasma, the plasma will actually antishield the test charge; the plasma will enhance the field of the test charge rather than diminish it.[2]

Antishielding is obviously very different from the three other shielding regimes; the distinction between Debye, instantaneous, and adiabatic shielding is less dramatic but nonetheless measurable; the three shielding regimes have different nonlinear properties, and result in different distribution functions. In this paper we will focus on instantaneous and adiabatic shielding, and illustrate the differences between these regimes analytically, experimentally, and computationally. Our discussion is limited to plasmas in which the electron motion is one dimensional. Such unidirectional motion could be enforced by the geometry of the plasma and the applied test charges, or it could be due to a strong magnetic field. We will also assume that ions, if present, do not move.

II. ANALYSIS

When a positive test charge is placed in a plasma, the electron orbits will be divided into two classes: a set of trapped orbits in the vicinity of the test charge, and a set of free-streaming orbits which pass the test charge.[6] The differences between Debye, instantaneous, and adiabatic shielding depend on how the trapped orbits are populated.[7] Indeed, there must be trapped electrons; if there are none, the plasma will antishield the test charge.[2,3] These distinctions are best illustrated with a test well rather than with a test charge (see Fig. 1). (A test well is an externally created potential that attempts, absent any self-consistent effects from the plasma itself, to bias the plasma positive or negative.) The orbits in a test well are shown in Fig. 2.

We can readily calculate the density of untrapped or free-streaming electrons by using the fact that v_0, the velocity of an electron far from the test well, is a constant of the motion. Any function of a constant of the motion is a solution of Vlasov's equation, thus the function $f(v)=f(v_0)=f_0(\sqrt{v^2-2\Phi})$, where $f_0(v)$ is the distribution function far from the well, and Φ is the self-consistent well depth, will be the distribution function of the untrapped electrons, i.e., those electrons with $|v|>\sqrt{2\Phi}$ in the well. (In this paper we normalize velocities by $v_{\mathrm{th}}=\sqrt{kT/m}$ and potentials by kT/e, where kT is the thermal energy, m is the electron mass, and e is the electron charge.) We can find the density of untrapped electrons by integration if we further assume $f_0(v)$ is a Maxwellian. Thus,

$$n_{\mathrm{Free}}(\Phi)=n_0\exp(\Phi)\mathrm{erfc}(\sqrt{\Phi}). \tag{1}$$

This result is the same for all shielding regimes. Since $n_{\mathrm{free}}(\Phi)$ is always less than one, the free-steaming electrons by themselves will antishield the test well; shielding requires trapped electrons.

The density of trapped electrons depends on the shielding regime. For Debye shielding, collisions will Maxwellian-

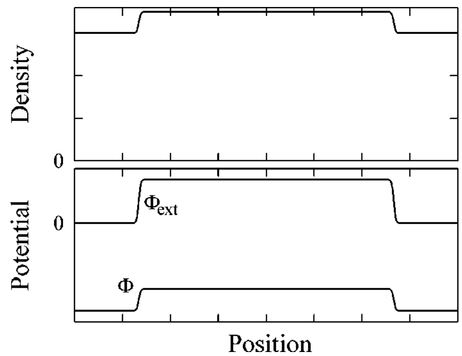

FIG. 1. Test well formed by applying an external potential Φ_{ext}, the resulting self-consistent potential Φ, and the resulting density n.

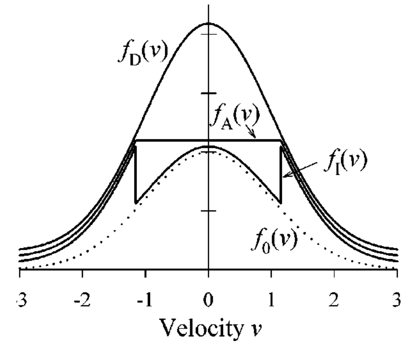

FIG. 3. Shielding distributions functions $f(v)$ for a well of depth $\Phi=2/3$. The distribution functions are offset for clarity.

ize the electrons in the test well, so the distribution function will be a Gaussian, as shown in Fig. 3. Integration of the distribution function out to the trapping velocity, $|v| = \sqrt{2\Phi}$, yields

$$n_{\text{DTrap}}(\Phi)=n_0 \exp(\Phi)\text{erf}(\sqrt{\Phi}). \tag{2}$$

The total Debye shielded density is the sum of the trapped and untrapped densities

$$n_{\text{Debye}}(\Phi)=n_0 \exp(\Phi). \tag{3}$$

For instantaneous shielding, only those electrons that happen to be in the well at the instant that the well is created and have velocity less than the trapping velocity will be trapped. Since instantaneously creating the well does not change the energy of these initially trapped electrons, their distribution function remains unchanged (see Fig. 3). Thus the trapped distribution function $f(v)$ will equal initial distribution function $f_0(v)$, and if $f_0(v)$ is a Maxwellian, the trapped density will equal

$$n_{\text{ITrap}}(\Phi)=n_0 \,\text{erf}(\sqrt{\Phi}). \tag{4}$$

The total density will be

$$n_{\text{Inst}}(\Phi)=n_0\{\exp(\Phi)+\text{erf}(\sqrt{\Phi})[1-\exp(\Phi)]\}. \tag{5}$$

Finally, in the adiabatic shielding regime, the test well is turned on so slowly that very low energy electrons are trapped as they transit the well because the well depth will be greater when the electrons try to leave the well than when

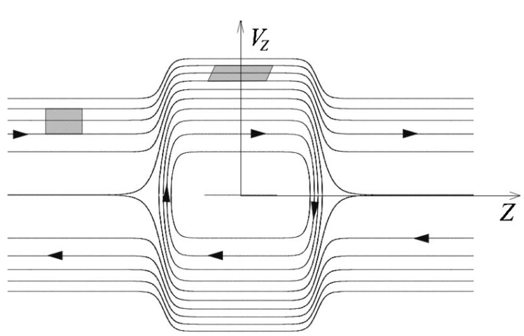

FIG. 2. Phase space orbits of electrons in a potential square well. Note the existence of both trapped and untrapped (free-streaming) orbits. The two shaded squares have the same area and visually demonstrate that the untrapped density is lower in the well than outside.

they entered it. In this case, the distribution function of the electrons will equal $f_0(0)$ (see Fig. 3), and the density of trapped electrons will be

$$n_{\text{ATrap}}(\Phi)=2n_0\sqrt{\frac{\Phi}{\pi}}. \tag{6}$$

The total density will be

$$n_{\text{Adiab}}(\Phi)=2n_0\sqrt{\frac{\Phi}{\pi}}+n_0 \exp(\Phi)\text{erfc}(\sqrt{\Phi}). \tag{7}$$

Expanding Eqs. (3), (5), and (7) for small Φ gives

$$n_{\text{Debye}}(\Phi)=n_0\left(1+\Phi+\frac{1}{2}\Phi^2+\cdots\right), \tag{8}$$

$$n_{\text{Inst}}(\Phi)=n_0\left(1+\Phi-\frac{2}{\sqrt{\pi}}\Phi^{3/2}+\frac{1}{2}\Phi^2+\cdots\right),$$

$$n_{\text{Adiab}}(\Phi)=n_0\left(1+\Phi-\frac{4}{3\sqrt{\pi}}\Phi^{3/2}+\frac{1}{2}\Phi^2+\cdots\right). \tag{9}$$

To order Φ, the densities are the same. However to next order ($\Phi^{3/2}$) both the instantaneous and the adiabatic densities are lower than the Debye density. Specifically,

$$n_{\text{Debye}}(\Phi)>n_{\text{Adiab}}(\Phi)>n_{\text{Inst}}(\Phi), \tag{10}$$

i.e., the instantaneous density is weaker than the adiabatic density. Since we expect that the shielding will be more complete the larger the response, this inequality implies that a collisionally shielded well will be better shielded than a well in which the plasma has no time to make a collision, and a well which is turned on slowly will be better shielded than a well that is turned on abruptly. Note that the potential Φ used in the above equations is the self-consistent potential: the sum of the external potential Φ_{ext} applied to make the well, and the potential that results from the response of the plasma.

These three different regimes are not sharply distinguished, but shade into each other continuously. For instance, assume that the well formation time is τ (normalized by the plasma frequency ω_p). In the instantaneous regime, all the electrons in the well with kinetic energy less than the self-consistent well depth Φ are trapped. But if the well size is L (normalized by the Debye length λ_D), electrons will be able to escape the well if $L \lesssim \sqrt{\Phi}\tau$. These escaped electrons

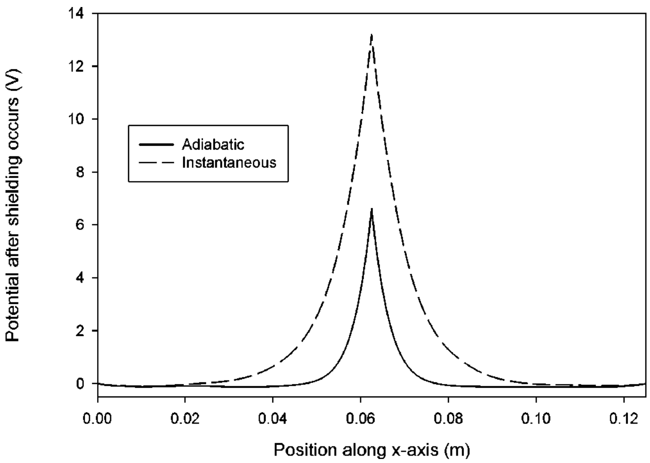

FIG. 4. Response to a large test charge plane placed at 0.06 m, calculated using the PDP1 program. The plasma had an initial density of 1×10^{13} m^{-3}, a temperature of 1.4 eV, and a Debye length of 0.004 m. Without the plasma, the test charge plane would have a potential of 100 V. The instantaneous shielded potential was found by turning the charge on abruptly, while the adiabatic shielded potential was found by turning the charge on over many electron transit times.

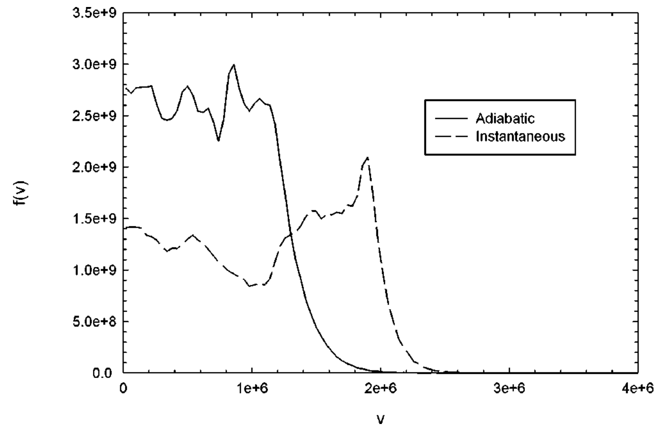

FIG. 5. Distribution functions from the PDP1 simulation. Here, $kT = 1.4$ eV, the Debye length is 0.004 m, and the distribution functions were measured five Debye lengths from the test plane. The distribution function resemble the predicted functions shown in Fig. 3: the adiabatic distribution function is roughly flat in the center, and the instantaneous distribution function has peaks near the trapping velocity.

will be replaced by adiabatically trapped electrons; the transition to the adiabatic regime will have begun. If, as would be the case of a test charge, L is on the order of the Debye length, and Φ is on the order of the temperature, than this relation reduces to $\lambda_D \sim v_{\text{th}}\omega_p$, i.e., the transition to the adiabatic regime begins when the well formation time is on the order of the plasma oscillation period. Similarly, the adiabatic regime will transition to the Debye regime once collisions redistribute the trapped population. This time is on the order of the collision time for the plasma, suitably reduced by the fact that the relevant trapped electrons may possess little energy.

III. ONE-DIMENSIONAL NUMERIC SIMULATIONS

We used the one-dimensional (1D) particle-in-cell (PIC) computer simulation PDP1 to model shielding in the instantaneous and adiabatic regimes. The simulation propagates electrons in a fixed positive background which neutralizes the initial electron background, and continuously injects Maxwellian-distributed electrons from both sides to model an infinite length plasma. We begin the simulation by allowing the electrons to reach an equilibrium, after which we switch on a transparent test potential grid in the center of the simulation. (The time scale on which we switch on the grid differentiates between the two regimes.) After the plasma has re-equilibrated, we measure the potential distribution and the distribution functions.

The simulations confirm that instantaneous shielding is less effective than adiabatic shielding (Fig. 4). The simulations also confirm (Fig. 5) that the distribution functions resemble the idealized distribution functions shown in Fig. 3. This is somewhat surprising because the instantaneous distribution function resembles a ‘‘bump on tail’’ distribution and is unstable according to the Penrose criterion[8] for spatially uniform plasmas. Nonetheless the bump remains after many tens of microseconds—many plasma oscillations. The distribution must be stabilized by finite length effects; the region where the distribution appears unstable is not much larger than a Debye length.

IV. EXPERIMENTAL RESULTS AND COMPARISON TO TWO-DIMENSIONAL SIMULATIONS

We also confirmed the predictions of the analytic theory with experiments with pure-electron plasmas. The plasmas were confined in a Malmberg–Penning trap.[9] Figure 6 shows a schematic of the trap, which consists of a series of electrically isolated hollow cylinders immersed a strong axial magnetic field. This field confines the plasma radially and causes the electrons to move one dimensionally. An axial potential well, created by biasing the trap cylinders, confines the plasma axially. The typical plasma parameters for this experiment are shown in Table I. We adjust the plasma temperature by heating the plasma by coupling it to a broadband noise source. The resulting plasma temperatures range from 1.6 to 10 eV. Since we wish to be in the collisionless regime, we make all our measurements in a time significantly shorter than a collision time.

We create a test well by biasing one of the trap cylinders. By controlling the rate at which we bias the cylinder we can create the well instantaneously or adiabatically. The transition time scale is the electron transit time through the cylinder, which is on the order of 200 ns. Our electronics can bias

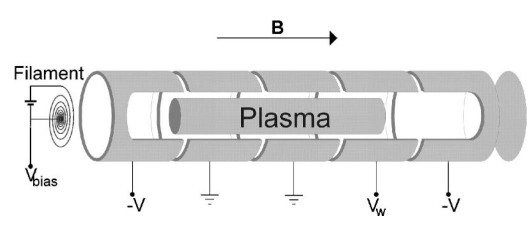

FIG. 6. Simplified experimental schematic. The plasmas are created by thermionic emission from the spiral tungsten filament, and are diagnosed by allowing them to flow through a movable pinhole onto the detector plate on the right.

TABLE I. Typical experimental parameters.

Magnetic field	B	500–1500 G
Density (peak)	n	$(1-4)\times 10^7$ cm^{-1}
Temperature	T	1.6–10 eV
Collision time $(e-e)$	$\tau_{e,e}$	1–200 ms
Collision time $(e-n)$	$\tau_{e,n}$	1–200 ms
Bounce time (average)	τ_b	300–800 ns
Plasma frequency (peak)	ω_p	30–50 MHz
Diocotron frequency	ν_D	40–200 kHz
Cyclotron frequency	ω_c	9–35 GHz
Debye length	λ_D	0.15–1 cm
Plasma column radius	r_p	0.7–1.3 cm
Plasma length	L_p	28 cm
Well length	L_w	14 cm

the cylinders in approximately 20 ns, comfortably in the instantaneous regime, or over hundreds of microseconds, easily in the adiabatic regime. We measure the response of the plasma by determining the charge in the test well, which we can measure by determining the image charge on the test well cylinder.[10]

We also performed a computer simulation to verify our experimental results. We use the two-dimensional (2D) $(r, z, 3v)$ PIC code OOPIC whose parameters we tailor to our experimental setup.[11] We can duplicate the physical dimensions, electron density, and plasma temperature. The most significant physical difference between the code and the experiment is that the code assumes a flat-topped radial plasma distribution while the actual distribution is rounded. In addition, the code assumes that the magnetic field is infinite.

Figure 7 plots the amount of charge in the well region, normalized to the charge in a zero-volt well, as a function of the well depth for both the adiabatic and instantaneous regimes. The figure shows both the results of the experiments and the corresponding OOPIC simulations. For small well depths the adiabatic and instantaneous cases are identical as we would expect because the linear response in the two regimes is identical. However, the amount of charge in the well is greater in the adiabatic regime than in the instantaneous regime when the well depth is large, as predicted by Eq. (9). Finally the computer simulations agree with the experimental results.

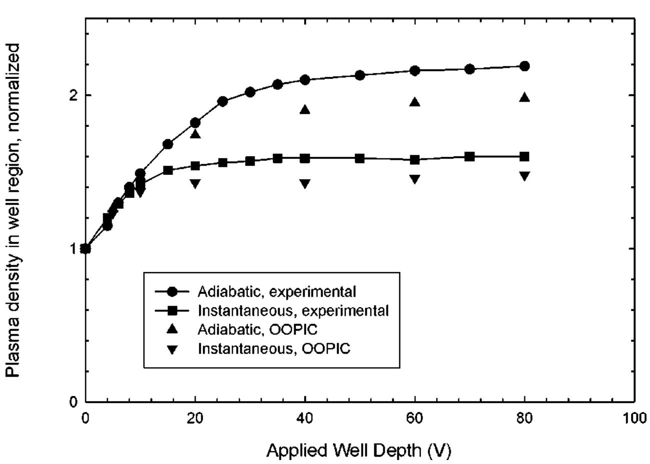

FIG. 7. Shielding as a function of applied well depth. The vertical axis reports the total amount charge in the well region, which includes both trapped and untrapped charge. The charge is normalized to the amount of charge in a 0 V well. We plot experimental and OOPIC simulation results for both adiabatic and instantaneous wells. The plasma temperature was 1.6 eV.

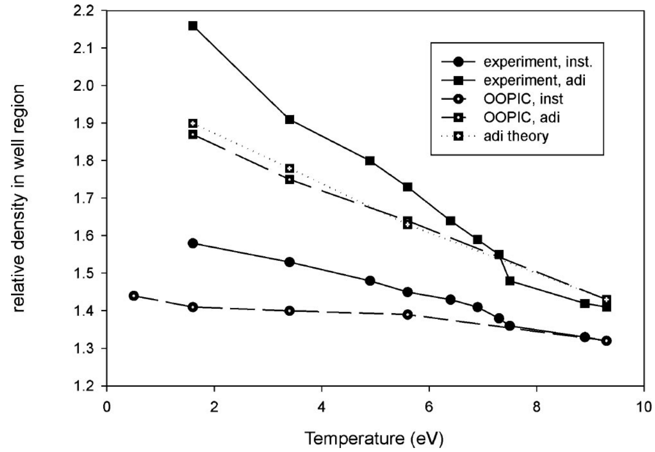

FIG. 8. Shielding as a function of temperature. We plot experimental and OOPIC simulation results for both adiabatic and instantaneous cases. The dotted line is from a finite-length trapping theory by Hansen (Ref. 4). The applied well voltage is 30 V.

Figure 8 plots the normalized charge in the well as a function of the plasma temperature with the well depth held fixed. The shielding improves in both regimes at low temperatures though the improvement is not large for the instantaneous regime. Once again, the computer simulations are in rough agreement with the data. The data also agrees with the prediction of a 1D, finite-length adiabatic regime theory by Hansen *et al.*[2,3]

V. CONCLUSIONS

We have shown through theoretical analysis, experiments, and computer simulations that shielding of an instantaneously applied perturbation differs from shielding of an adiabatically applied perturbation in one-dimensional, collisionless plasmas. For small perturbations, the plasma response is nearly the same in both cases, but for large perturbations, an adiabatic well is shielded more strongly than an instantaneous well. Furthermore, simulations show that the distribution of particle velocities in the well roughly agrees with predictions.

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