The VME and its Non-Relativistic Avatar the GVME

UC Berkeley, March 30, 2019

Niels Obers (Nordita & Niels Bohr Institute)

Marty Halpern Memorial:

From mesons to orbifolds via affine Lie algebras

History

Met Marty first as grad student in QFT course (1987)

Reading course on ST

Collaborated from then untill 2002 (last paper with Marty, also with Ori/Craig)

Taught me a great deal of physics & sociology of academia

Memorable summervisits to CERN and Copenhagen

Introduced me to Niels Bohr Institute !

146 papers in wide variety of topics

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PHYSICAL REVIEW D VOLUME 3, NUMBER 10 15 MAY 1971

New Dual Quark Models*

K. BARDAKCI AND M. B. HALPERN
Department of Physics, University of California, Berkeley, California 94720
(Received 16 November 1970)

On the basis of new representations of the projective group, we construct some new dual quark models whose spin and internal symmetry are not multiplicative. One model is a factorized theory of exotic states with broken exchange degeneracy, ninth mesons being optional. The exotic states are suppressed three units below the Pomeranchon. In another model, with spin-orbit coupling and curved trajectories, both spin ghosts and orbital ghosts are involved in the Ward identities.

Nuclear Physics B250 (1985) 689–715

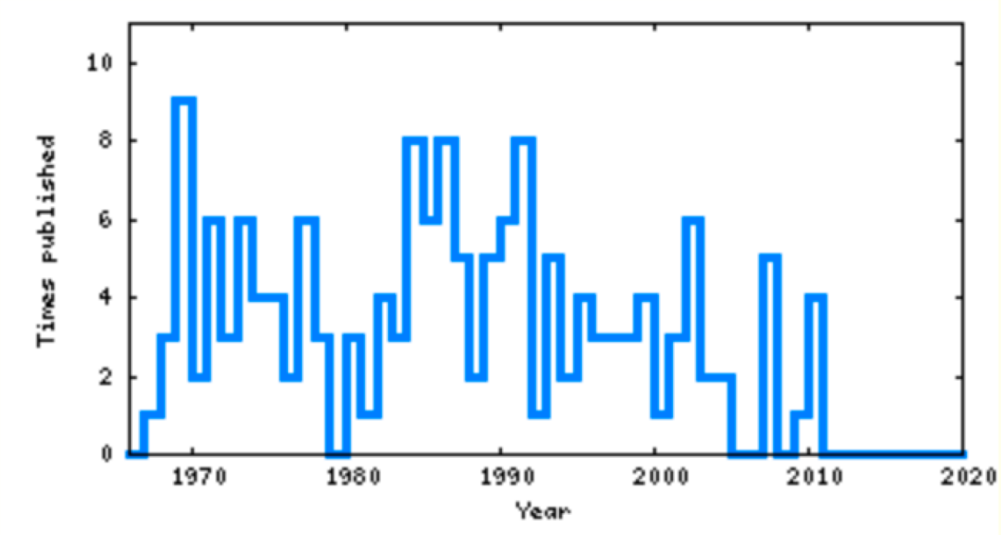
SUPERSYMMETRIC GROUND STATE WAVE FUNCTIONS

Mark CLAUDSON* and Martin B. HALPERN*
Department of Physics, University of California, Berkeley, California 94720, USA

Received 23 May 1984
(Revised 7 November 1984)

The construction of explicit supersymmetric ground states is considered in a variety of quantum mechanical systems. For broad classes of supersymmetric hamiltonians it is not difficult to find closed-form zero-energy ground-state wave functions.

Publication Graph



+

String Theory  2D CFT

$$T(z)T(w) \sim \frac{c}{(z-w)^4} + \left[\frac{2}{z-w} + \frac{\partial_w}{z-w}\right] T(w)$$

Virasoro

What is the space of CFTs that have an underlying affine Lie algebra structure ?

$$J_a(z) J_b(w) \sim \frac{i f_{ab}{}^c}{z-w} J_c(w) + \frac{k \eta_{ab}}{(z-w)^2}$$

structure constants

level

Known constructions from70s-80s

for history see: Bardakci,Halpern in ``The birth of String Theory''

affine-Sugawara construction

$$T_g(z) = \frac{\eta^{ab}}{2k+c_g} :J_a J_b:(z)$$

$$c_g = \frac{k \dim g}{2k+c_g}$$

$\sim$ dual Coxeter #

WZW model

& coset construction
(gauged WZW model)

$$T_{g/h} = T_g - T_h$$

$$c_{g/h} = c_g - c_h$$

$h^\vee c_g$

affine-Virasoro construction (Halpern,Kiritsis (1989))

$$T(z) = L^{ab} \, {:}J_a J_b{:}(z) \quad (+ D^a \partial J_a)$$

Q: under what conditions is T(z) Virasoro ?

→ Virasor Master Equation (VME)

$$L^{ab} = 2L^{ac}G_{cd}L^{db} - L^{cd}L^{ef}f_{ce}{}^{a}f_{df}{}^{b} - L^{cd}f_{ce}{}^{f}f_{df}{}^{(a}L^{b)e}$$

$$c = 2G_{ab}L^{ab}$$

- set of coupled 2nd order algebraic equations
- generically solutions with irrational central charge/conformal weights

ICFT ⊃ RCFT

Martyisms

…. does not know how to punch his way through a paper bag..

…… should be shot at dawn…

(Niels) , this is just a morass of algebra…

it is always good to have a baseball bat in the car, just in case…

… & many more
(unfortunately forgotten)

ICFT see review: Halpern,Kiritsis,NO,Clubok, (Phys Rep. 1995)

numerous developments:

- affine-Virasoro space:
  many solutions, consistent ansatze,
  high-level perturbation theory,
  connections with (generalized) graph theory

- superconformal, W-algebra generalizations

- C-function (see talk Elias)

- Dynamics: actions, ICFT on sphere & torus,
  high-level conformal blocks

Important property: K-conjugation & nested cosets

solutions come in
K-conjugate pairs

$$T + \bar{T} = T_g$$

$$T(z)\,\bar{T}(w) \sim \text{reg}$$

nested cosets (new RCFTs ? role in non-trivial RG fixed points ?)

$$g \supset h_1 \supset h_2$$

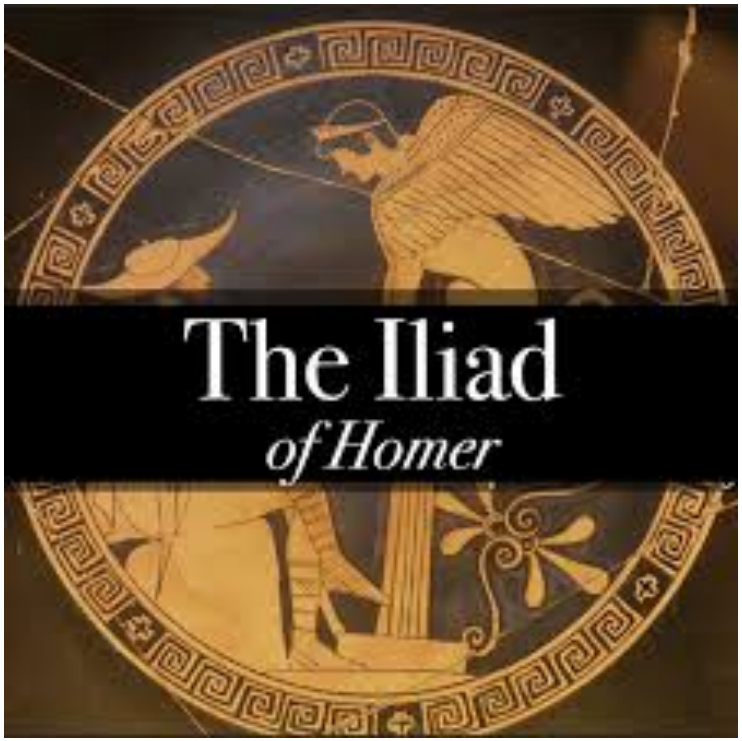
$$T_{g/h_1/h_2} + T_{h_1/h_2} = T_g$$

$T_{g/h_1/h_2}$: nested cosel.

T_{h_1/h_2}: regular cosel

$$c = c_g - c_{h_1} + c_{h_2}$$

Marty and extremes

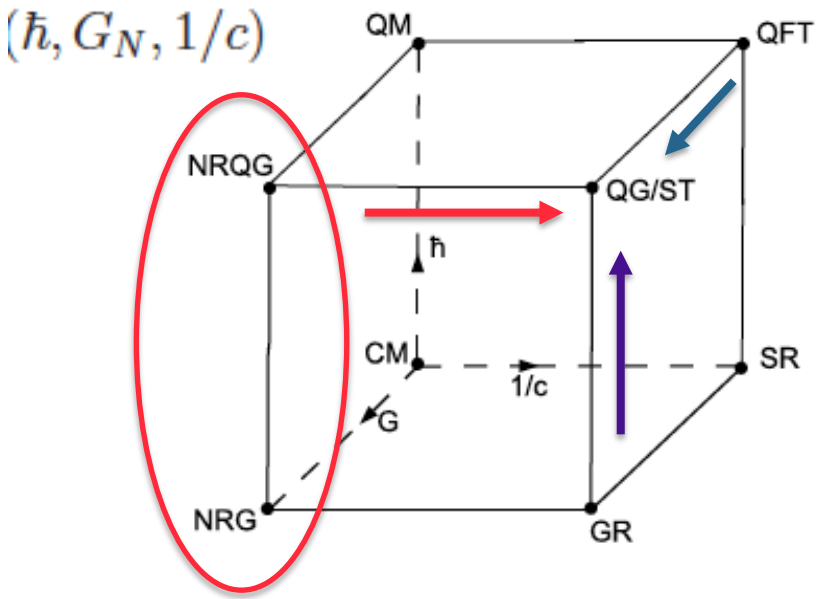


Tribute to Marty (in progress)

- new algebraic equation
- new abbreviation

GVME → Galilean

Motivation: cube of physical theories



a third route towards
(relativistic) quantum gravity

how does this fit with
string theory/holography ?

already (classical) non-relativistic gravity (NRG)
is more than just Newtonian gravity

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Theories at Infinite Momentum*†

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(Received 25 July 1968)

We construct Galilean-invariant theories (with Schrödinger equations) at infinite momentum that describe interacting relativistic systems. Classes of both first- and second-quantized theories are presented. The formalism provides a general approach to the saturation of current algebra; positivity of the mass spectrum is guaranteed, and as much inelasticity as necessary may be introduced. More generally, however, such theories offer the hope of potential-theoretic intuition for relativistic physics.

Strings on torsional Newton-Cartan geometry

- null-reduction of relativistic point particle gives action of (massive) non-relativistic point particle coupling to (torsional) Newton-Cartan geometry

→ generalize to: null reducing Polyakov action [Harmark,Hartong,NO](1705) [Harmark et al](1810)

TNC fields can be uplifted to Lorentzian metric with null Killing isometry :

$$ds^2 = G_{MN}dx^M dx^N = 2\tau(du - m) + h_{\mu\nu}dx^\mu dx^\nu$$

$$\mathcal{L}_{\rm Pol} = -\frac{T}{2}\Big[\, 2\epsilon^{\alpha\beta}m_\alpha\partial_\beta\eta + e\,\eta^{ab}e^\alpha{}_a e^\beta{}_b h_{\alpha\beta} + \lambda_+\epsilon^{\alpha\beta}(e_\alpha{}^0 + e_\alpha{}^1)(\tau_\beta + \partial_\beta\eta) + \lambda_-\epsilon^{\alpha\beta}(e_\alpha{}^0 - e_\alpha{}^1)(\tau_\beta - \partial_\beta\eta)\Big]$$

- on flat target space → Gomis-Ooguri (2000) non-relativistic string

take "zero tension" limit with rescaling to keep action finite:

$$\mathcal{L}_{\rm NRPol} = -\frac{T}{2}\Big[2\epsilon^{\alpha\beta}m_\alpha\partial_\beta\eta + e\,e^\alpha{}_1 e^\beta{}_1 h_{\alpha\beta} + \omega\epsilon^{\alpha\beta}e_\alpha{}^0\tau_\beta + \psi\epsilon^{\alpha\beta}\Big(e_\alpha{}^0\partial_\beta\eta + e_\alpha{}^1\tau_\beta\Big)\Big]$$

flat world-sheet gauge: residual symmetry = GCA

from (double, i.e. target space/world-sheet) non-relativistic limit of Polyakov action:

$S_{pol} \longrightarrow S_{NR}$

Vir × Vir GCA $(= BMS_3)$

$$[L_n\,, L_m] = (n-m)L_{n+m}\,, \qquad [L_n\,, M_m] = (n-m)M_{n+m}\,.$$

- strings moving in non-relativistic target space (Newton-Cartan like)
- world-sheet theory is also non-relativistic

→ novel class of sigma-models with GCA symmetry

- WS theories directly related to near-BPS limits of AdS/CFT

dual: quantum mechanical theory giving spin chains in large N limit

simplest example: Landau-Lifsthiz model from SU(2) spin chains

realizations of GCA on Galilean affine Lie algebras ?

Galilean contraction of Vir x Vir

$\mathcal{L}_m \quad \bar{\mathcal{L}}_m$

$$L^0_m \equiv L_m = \mathcal{L}_m + \bar{\mathcal{L}}_m$$
$$L^1_m \equiv M_m = \epsilon\,(\mathcal{L}_m - \bar{\mathcal{L}}_m) \qquad \epsilon \to 0$$

satisfies GCA

similarly from two copies of affine Lie algebra:

→ Galilean affine Lie algebra

$$[\,J_a^{0(m)}, J_b^{0,(n)}\,] = f_{ab}{}^c\, J_c^{0,(m+n)} + k_0\,\eta_{ab}\,m\,\delta_{m+n,0}$$
$$[\,J_a^{0(m)}, J_b^{1(n)}\,] = f_{ab}{}^c\, J_c^{1(m+n)} + k_1\,\eta_{ab}\,m\,\delta_{m+n,0}$$
$$[\,J_a^{1(m)}, J_b^{1(n)}\,] = 0$$

- can be generalized to multiple copies (via ``IW bundle")

Galilean affine Sugawara construction ?

require
(see Rasmussen,Raymond (2017,2019))

$$T_0 J_0 \sim J_0 \quad \text{weight 1 primary}$$
$$T_0 J_1 \sim J_1$$
$$T_1 J_0 \sim J_1$$
$$T_1 J_1 \sim \text{reg.}$$

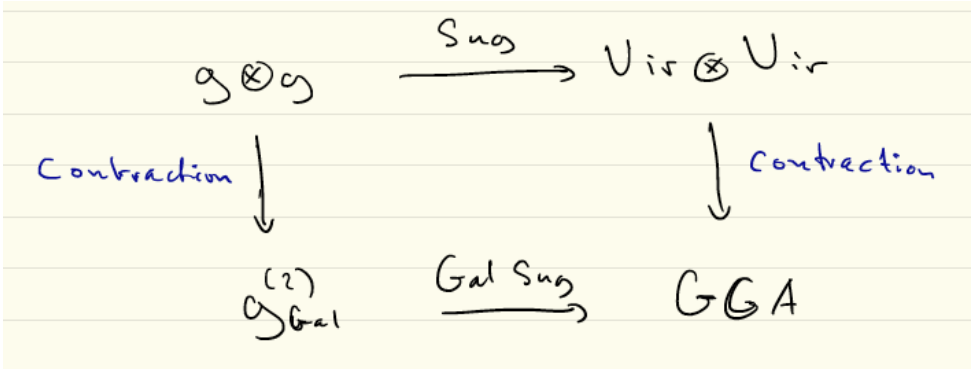
satisfied by:

$$T_0 = \frac{\eta^{ab}}{2k_1} :J^0_a J^i_b + J^1_a J^0_b: - \frac{k_0 Q_g}{2k_1^2} :J^1_a J^k_b:$$

$$T_1 = \frac{\eta^{ab}}{2k_1} :J^1_a J^1_b:$$

$$c_0 = 2\dim g \qquad c_1 = 0$$

contractions and
Sugawara commute:



Galilean Virasoro Master Equation (GVME)

construct:

$$T_i = \sum_{s,r=0}^{i} L^{ab}_{i;(r,s)} \, {}^{\times}_{\times} J^{r}_{a} J^{s}_{b} {}^{\times}_{\times} \qquad i=0,1$$

& require GCA

take (for simplicity):

$$T_0 = L^{ab} \, {}^{\times}_{\times} J^{0}_{a} J^{1}_{b} + J^{1}_{a} J^{0}_{b} {}^{\times}_{\times} + \Pi^{ab} \, {}^{\times}_{\times} J^{1}_{a} J^{1}_{b} {}^{\times}_{\times}$$

$$T_1 = N^{ab} \, {}^{\times}_{\times} J^{1}_{a} J^{1}_{b} {}^{\times}_{\times}$$

→

GVME

$$\begin{cases} L^{ab} = 2k_0 L^{ac} \eta_{cd} L^{db} \\ \Pi^{ab} = 2k_0 L^{ac} \eta_{cd} L^{db} + 2 L^{ce} L^{df} f_{cd}{}^{a} f_{ef}{}^{b} \\ \qquad\qquad + 4 k_1 L^{ac} \eta_{cd} \Pi^{db} \\ N^{ab} = 2k_1 L^{ac} \eta_{cd} N^{db} \end{cases}$$

New rational solutions and WS actions

- check: Galilean affine-Sugawara construction is solution
- new rational solution: analogue of coset construction, Galilean cosets

… (more ?)

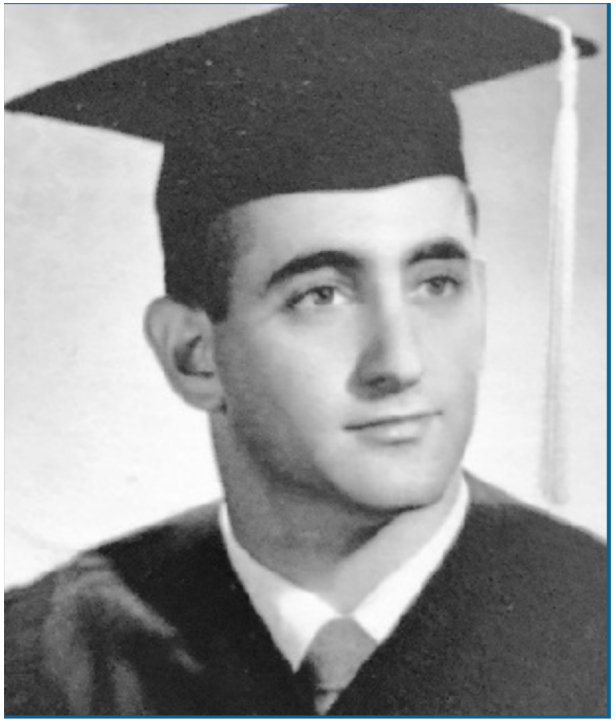
→ What are the corresponding world-sheet actions ?

Galilean WZW actions (and gauged WZW)

- apply same contraction limit at action level (in progress)

- important connection to the new class of GCA sigma models & studying their quantization
- expected to correspond to exact versions of the these non-relativistic sigma models

Marty's legacy lives on in modern developments of string theory/holography/gravity



Thank you all for coming to the meeting !