nature physics

REVIEW ARTICLES

PUBLISHED ONLINE: 30 OCTOBER 2017 | DOI: 10.1038/NPHYS4302

The physics of quantum materials

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The physical description of all materials is rooted in quantum mechanics, which describes how atoms bond and electrons interact at a fundamental level. Although these quantum effects can in many cases be approximated by a classical description at the macroscopic level, in recent years there has been growing interest in material systems where quantum effects remain manifest over a wider range of energy and length scales. Such quantum materials include superconductors, graphene, topological insulators, Weyl semimetals, quantum spin liquids, and spin ices. Many of them derive their properties from reduced dimensionality, in particular from confinement of electrons to two-dimensional sheets. Moreover, they tend to be materials in which electrons cannot be considered as independent particles but interact strongly and give rise to collective excitations known as quasiparticles. In all cases, however, quantum-mechanical effects fundamentally alter properties of the material. This Review surveys the electronic properties of quantum materials through the prism of the electron wavefunction, and examines how its entanglement and topology give rise to a rich variety of quantum states and phases; these are less classically describable than conventional ordered states also driven by quantum mechanics, such as ferromagnetism.

The way we think about manifestations of quantum physics in materials has recently undergone a profound change of perspective. Although materials scientists and engineers have long exploited quantum effects in a range of electronic devices—well-known examples are the quantized electronic energy levels and optical selection rules at the heart of optoelectronics, and the tunnel effect that underlies the upcoming generation of hard-disk drives[1,2]—the past decade has seen a dramatic increase in our understanding of how subtle quantum effects control the macroscopic behaviour of a whole range of different materials.

Two strange and beautiful aspects of quantum mechanics have come to the fore. One is the topological nature of quantum wavefunctions. A familiar example is the existence of quantized vortices in superconductors. These vortices exist because of the requirement that the superconducting condensate have a well-defined phase, and gauge invariance fixes how this phase couples to magnetic flux. The phase can wind only by an integer multiple of 2π around a vortex, and this integer winding number is a simple example of a topological invariant: a quantity that remains fixed under smooth changes of a system. Similar topological quantities turn out to govern many other kinds of materials, not just superconductors, and these support phenomena ranging from dissipationless transport to novel quasiparticle excitations.

Another deep feature of quantum mechanics is the non-local entanglement of some quantum states that is spectacularly highlighted in teleportation experiments with two photons separated over macroscopic distances[3]. Even the wavefunction of two spins in a singlet is entangled, in that the wavefunction of either spin by itself is not well defined. In the words of Schrödinger[4], who coined this term: "Another way of expressing the peculiar situation is: the best possible knowledge of a whole does not necessarily include the best possible knowledge of all its parts." Thinking about entanglement in a solid is mind-boggling even in seemingly simple materials: the states of all $\sim 10^{23}$ electrons in a typical chunk of metal are superposed in such a way that the many-body wavefunction covering the entire solid changes sign whenever two electrons are interchanged, as required by their fundamental nature as fermions.

The Fermi statistics forcing this specific, antisymmetric entanglement also implies that only electrons with energies close to the Fermi level contribute significantly to the transport and thermodynamic properties of metals. However, this still leaves a massive number of such 'conduction electrons' whose wavefunctions are intricately entangled. Quantum materials can be defined as those with novel entanglement or topological properties—that is, materials with entanglement beyond the requirement of Fermi statistics and with topological responses such as the vortex formation described above. For example, the entanglement between spins in complex magnets, or electrons in a Cooper pair, is an important part of how these fail to be captured by conventional pictures (we prefer this definition to simply invoking strong correlations or strong interactions, because, for example, a Fermi liquid renormalized by strong interactions may have very different correlations from a conventional metal but is ultimately in the same phase—they are, so to speak, 'adiabatically connected'). Topology and entanglement lead to new kinds of quantum order that are sharply distinguished from conventional states by the existence of phase transitions. Today's materials science has barely scratched the surface of these remarkably complex quantum states of matter.

While Fermi statistics is a basic and immutable property of electronic wavefunctions, a feature of many classes of quantum materials is the emergence of new kinds of particles—quasiparticles—whose properties may be rather different from those of the underlying electrons. Again superconductors provide a ready example: the Bogoliubov quasiparticles of a superconductor are complex superpositions of electron and hole without a well-defined electric charge. These quasiparticles are still fermions, but topological quantum materials support other kinds of emergent quasiparticles with new kinds of exchange statistics. Similarly, there is increasing experimental evidence that in some bulk magnets the conventional spin-wave excitation, known as a magnon, breaks up

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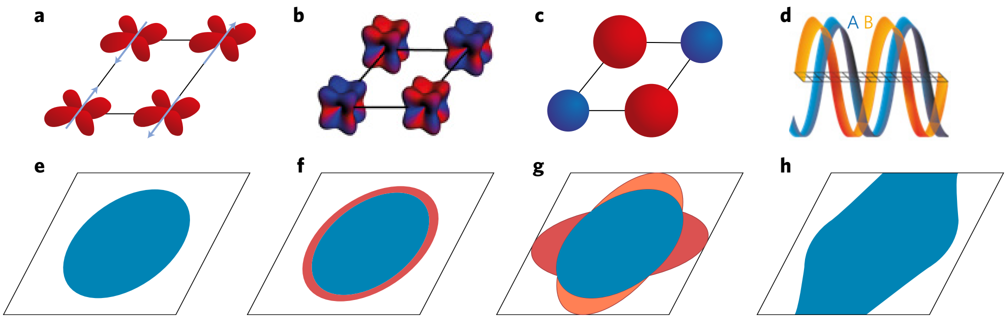

Figure 1 | Collective order of electrons on a two-dimensional square lattice. Black lines in **a**–**d** indicate the crystallographic unit cell in real space, black lines in **e**–**h** indicate the first Brillouin zone in momentum space. **a**, Uniform *d*-orbital order (red) and antiferromagnetic spin order (blue) in 3*d* electron systems. **b**, Antiferromagnetic order of spin–orbit isospins in 4*d* and 5*d* electron systems. The colours indicate the admixture of different *d*-orbitals to the spin–orbit-entangled wavefunction. **c**, 'Chequerboard' charge order. The colours indicate different valence states of the constituent metal ion. **d**, 'Intertwined' state of different order parameters A and B. Examples are the 'striped state' for A = uniform metal (**e**) and B = antiferromagnetism (**a**), and the 'pair density wave' for A = uniform metal and B = superconductivity (**f** or **g**). **e**, Uniform metallic state. The blue line indicates the Fermi surface, and the blue shading indicates filled electron states. **f**, *s*-wave superconductivity. The superconducting gap is indicated in red. **g**, *d*-wave superconductivity. The different shadings of red indicate different signs of the superconducting gap function. **h**, Nematic order. The open Fermi surface is a hallmark of the nematic state which spontaneously breaks the four-fold rotational symmetry of the electron system without breaking the translational symmetry.

into spinon excitations whose existence was previously only established in one-dimensional materials.

Of course there are good reasons for our failure to notice (let alone take advantage of) the exotic quantum beauty lurking in the materials that surround us. Scattering from random defects or thermally excited lattice vibrations scrambles the phase of the electrons' quantum wavefunction. However, unique quantum properties become apparent even in an ordinary metal such as copper upon cooling to temperatures of a few kelvin, where inelastic scattering between electrons and lattice vibrations is frozen out. If an external magnetic field then forces the electrons into circular orbits, various physical observables exhibit magnetic-field-dependent quantum oscillations that have no classical analogue[5]. A more spectacular manifestation of quantum entanglement is the superconducting state that appears in other elemental metals (including aluminium and lead) at comparably low temperatures. Here, an effective attraction between electrons mediated by lattice vibrations triggers a reorganization of the many-electron wavefunction into a phase-coherent superposition of Cooper pairs[6]. Although this re-entanglement affects only a small fraction of the electron states with energies close to the Fermi level, its stunning macroscopic manifestation—dissipationless charge transport—provides a glimpse of the potential of quantum many-body physics for real-world applications. Similar dissipationless transport, now associated with exceptionally precise quantization of observables such as the Hall conductance or Josephson frequency quantum, can also arise for topological reasons.

The dramatic expansion of quantum materials research began in 1986 with the discovery of high-temperature superconductivity in copper oxides[7], and with the consequent realization that superconductivity and other macroscopic quantum phenomena (beyond simple ferromagnetism) need not be limited to extreme conditions. This discovery emboldened the community to finally tackle one of the grand challenges of modern physics: understanding the influence of Coulomb interactions between conduction electrons, which reach deep below the Fermi level and massively entangle both spin and spatial components of the single-electron quantum states. The intellectual challenge and the prospect for novel applications sparked an enormous research effort that has branched out into many different fronts. Today's quantum materials include such unlikely candidates as hydrogen sulfide, a foul-smelling gas under ambient conditions that becomes a superconductor with a record-setting transition temperature exceeding 200 K under high pressure[8], and diamond, where entanglement of electronic and nuclear spins at defect centres enables quantum oscillations with coherence times of several seconds at room temperature[9,10].

A similarly dramatic expansion took place more recently in research on topological quantum materials. The notion that there could be a new kind of order, based upon the nontrivial topology of electron wavefunctions, emerged in the 1980s through studies of two-dimensional systems under extreme conditions of low temperature and strong magnetic field. The topological tools developed at that time, after suitable generalization to the case of zero field, led theorists two decades later to predict that even bulk materials could be in topological states—the 'topological insulators'. These also have signatures in terms of the entanglement of their electronic wavefunctions: entanglement in a bulk wavefunction effectively diagnoses whether a new metallic edge state will emerge at a boundary.

Many other topological states are now being found, including topological semimetals and strongly correlated topological states. Theorists have invented models with 'quantum liquid' ground states that are much more intricately entangled than those of ordinary metals and superconductors, and experimentalists are on the verge of identifying realizations of such states in actual quantum materials[11]. There is a subtle topology-dependent piece of long-ranged entanglement in these liquids that differentiates them from more conventional phases. So for both topological states and superconducting states, entanglement is a useful lens to isolate part of what makes them different from ordinary matter, and the intersection between superconductivity and topology is one of the most active areas at present.

This brief review cannot do justice to these diverse facets of current quantum materials research[12]. Rather, we focus on two generic aspects of electronic quantum states that generate phenomena qualitatively distinct from the simpler quantum effects utilized in today's electronics: correlations due to Coulomb interactions between electrons that promote entanglement beyond the one resulting from Fermi statistics alone, and the topological properties of single-electron Bloch wavefunctions in solids. These phenomena may well empower a new era in which quantum materials are broadly harnessed for device applications[13,14].

Quantum collective phenomena

The condensation of interacting electron systems into ground states with various forms of collective order has been an important subject of solid state research for many years. Before delving into some of the latest developments, it is useful to recall some basic aspects of electronic correlations that are already apparent in the physics of small molecules. The simplest example is the ground-state wavefunction of the hydrogen molecule, which is almost entirely composed of the degenerate 1*s*-orbitals and spin-1/2 states of the constituent atoms. The ground state is a spin-singlet with a symmetric combination of atomic orbitals that accumulates negative charge between the nuclei and thus minimizes their Coulomb repulsion. The competition with the spin-triplet state, which keeps the electrons apart and minimizes their mutual Coulomb repulsion, is quite subtle, but this problem could be solved with the limited computational tools available in the early days of quantum chemistry[15]. The singlet molecular wavefunction cannot be written as a product of single-electron states, thus illustrating that correlations generically entangle spin and orbital degrees of freedom. Remarkably, the ground state of the beryllium dimer with a total of eight electrons is already at the limit of today's much more powerful computers[16]. Although the Be_2 molecule is somewhat anomalous because correlations between electrons in the fully occupied, nearly degenerate 2*s* and 2*p* valence orbitals of the Be atoms are particularly pronounced, this simple example shows that the computational power required to analyse correlated-electron systems grows extremely rapidly with the number of electrons.

Solid state scientists have beaten these odds through many years of research with continuous back and forth between experiment and theory. As a result, the field has progressed from a qualitative understanding of electronic ordering phenomena to a point where theory-guided manipulation and design of specific properties resulting from electronic correlations are becoming more and more realistic. This development spans a wide range of materials, including Mott insulators where the Coulomb correlations are so strong that electrons remain tightly localized around atomic sites, and metals where electrons are highly delocalized and Coulomb correlations are effectively screened; some forms of electronic order are displayed in Fig. 1. Already in the 1950s, for instance, researchers had worked out a set of semi-empirical rules (now known as Goodenough–Kanamori rules) for the sign and magnitude of the effective spin–spin interactions in Mott insulators[17,18]. Today, the magnetic interactions and magnetic ordering patterns of common transition metal compounds with localized electrons in the 3*d* atomic shell (including, for instance, the prototypical Mott-insulator $LaMnO_3$) can be reliably computed using *ab initio* methods, which include a comprehensive set of single-electron quantum states and treat the electronic correlations in an approximate, but increasingly accurate manner[19]. *Ab initio* methods now also yield accurate values for the transition temperatures of conventional superconductors with weakly correlated electrons, ranging from Pb to MgB_2 (refs 20,21).

Novel magnets. However, even these seemingly mature areas of quantum materials research have recently seen surprising developments. These include the discovery that Mott insulators with 4*d* and 5*d* valence electrons exhibit an entirely different type of magnetic interaction than their well-known 3*d*-electron counterparts. In these materials, the relativistic spin–orbit coupling of the valence electrons is comparable to the correlation strength, so that the magnetic moments arising from the electrons' spin and orbital motion are firmly locked (Fig. 1b). The resulting spin–orbit-entangled wavefunctions give rise to magnetic interactions that are highly frustrated[22] even in the simple honeycomb lattice architectures realized in stoichiometric quantum materials such as $RuCl_3$, Li_2IrO_3 and Na_2IrO_3 (refs 23–25). Figure 2 shows resonant X-ray scattering data that directly demonstrate the presence of such interactions in a honeycomb iridate. As frustration inhibits the formation of conventional magnetic order, spin–orbit entanglement provides a new route towards the realization of model Hamiltonians with 'spin liquid' ground states—solid-state analogues of liquid helium where quantum fluctuations obliterate crystalline order even in the zero-temperature limit (see below).

In the course of the past decade, magnetic Mott insulators have also developed into a model platform to investigate electronic correlations near quantum phase transitions which are driven by an external control parameter at zero temperature. Figure 3 shows two examples of quantum transitions that separate conventional antiferromagnetic order from non-magnetic ground states composed of singlets. At the transition between these two very different forms of order, the ground-state wavefunction is entangled over macroscopic length scales. Recent neutron scattering experiments have revealed the emergence of new bosonic quasiparticles at magnetic quantum-critical points[26–28]. Ordinary magnets exhibit well-defined spin waves that modulate the direction of the magnetization, but the longitudinal modes that modulate its amplitude (roughly analogous to the Higgs modes found in superconductors and in particle physics[29]) are usually found at much higher energies and are strongly mixed with multimagnon excitations. Near the quantum-critical point, however, the longitudinal magnons become soft, and the neutron data demonstrate that they are protected against decay into transverse spin waves in some regions of momentum space. The Higgs mode and its dynamics can thus be studied in a new condensed-matter setting. The quantitative confrontation of neutron scattering data and theoretical work on insulating model magnets (Fig. 3) provides a firm basis for research on the influence of quantum-critical correlations on the properties of a much wider class of quantum materials, including metals and superconductors.

Excitonic insulators. Research on metals with highly delocalized, weakly correlated electrons has also yielded surprising recent discoveries. One of the highlights is a set of experiments on semimetals, including $TiSe_2$ and Ta_2NiSe_5, which undergo phase transitions into insulating ground states upon cooling[30–32]. Experimental evidence suggests that this transition is induced by the formation of excitons due to Coulomb attraction between electrons and holes in small pockets near the Fermi level. The condensation of excitons into an insulating many-body ground state is formally analogous to the superconducting transition triggered by the formation of Cooper pairs between two electrons—but it results in zero rather than infinite conductivity in the zero-temperature limit[33]. Recent experiments have provided evidence of a hidden connection between these two antagonistic ordering phenomena by showing that modest hydrostatic pressure or doping induce a superconducting phase in $TiSe_2$ (refs 34–36). This observation raises the question to what extent superconductivity in $TiSe_2$ and related materials can be described in terms of the standard theory of lattice-mediated attraction between electrons, or whether collective electronic modes due to Coulomb correlations play an essential role. Related investigations are now exploring whether superconductivity in $SrTiO_3$, which has long been regarded as a conventional electron–phonon superconductor, is instead driven by quantum-critical ferroelectric soft modes[37] or plasmons[38]. New opportunities to test these predictions arise from the emerging ability to control the dimensionality and density of the electron system in $TiSe_2$, $SrTiO_3$ and other complex materials in heterostructures[39,40] and exfoliated layers[41].

Unconventional superconductors. Research following the discovery of high-temperature superconductivity in iron pnictides[42] and chalcogenides[43] has created a model platform for the systematic exploration of electronically driven superconductivity and its interplay with different forms of electronic order[44]. In these materials, the correlation strength is moderate—that is, comparable to the single-electron bandwidth but not large enough to induce Mott localization. Electronic correlations manifest themselves in ubiquitous, strong antiferromagnetic spin fluctuations[45] whose key influence on superconductivity is documented by their strong

NATURE PHYSICS DOI: 10.1038/NPHYS4302

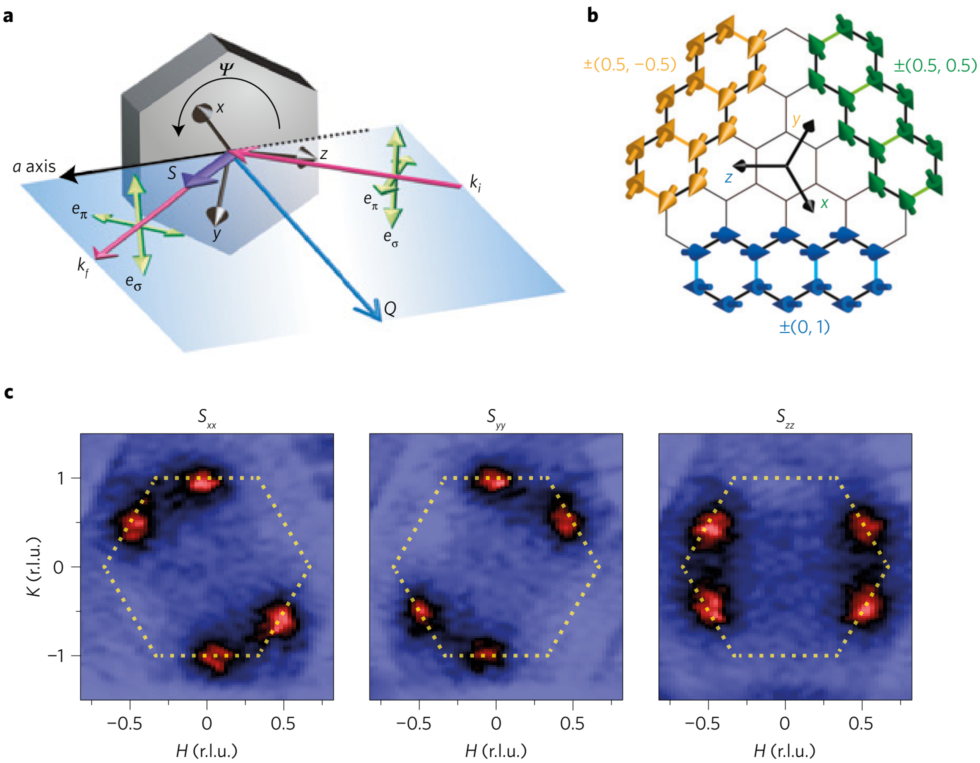

Figure 2 | Resonant X-ray scattering from a Kitaev magnet. a, Polarized X-rays (magenta) are scattered froma crystal of Na_2IrO_3, a material in which the magnetic Ir ions are arranged on a honeycomb lattice. The photon energy is tuned to an absorption edge of the magnetic Ir ions, and the scattered intensity is monitored as a function of the photon polarization (green) and wavevector **Q** (blue). **b**, Three fluctuating antiferromagnetic domains, with different directions of the Ir magnetic moments (arrows) and ordering wavevectors (indicated in the legend in reciprocal lattice units, r.l.u.). **c**, Intensity maps of the scattered X-rays as a function of two **Q** components of Cartesian coordinates. The data were recorded in polarization geometries sensitive to magnetic correlations, *S*, in the *x*, *y* and *z* directions. The correlation between the magnetic moment directions and the ordering wavevectors indicates predominant bond-directional (Kitaev) exchange interactions. Kitaev interactions are highly frustrated and can generate a spin liquid ground state even in simple lattice geometries. Adapted from ref. 25, Macmillan Publishers Ltd.

reaction to the onset of superconductivity, and by a sign reversal of the superconducting order parameter in Fermi surface regions that are strongly affected by spin fluctuations. Spurred in part by experimental observations on iron-based superconductors, the past years have seen rapid advances in theoretical methods—such as the functional renormalization group[46,47]—that treat moderately correlated-electron systems in an unbiased manner and with a minimum of uncontrolled approximations. Although these methods do not yet yield quantitative predictions of the superconducting transition temperatures, and many important questions (notably the role of lattice vibrations in driving superconductivity) remain to be answered, there are good reasons to expect continuous progress through systematic optimization of the material quality and the experimental and theoretical methodology, analogous to research on conventional magnets and superconductors discussed above.

Despite these advances, the understanding of strongly correlated electron systems near the Mott metal–insulator transition remains one of the greatest challenges in modern physics. The transition leads to a massive re-entanglement of the many-electron wavefunction, involving both spin and spatial degrees of freedom and reaching all the way to the bottom of the conduction band[48]. Even the simplest model designed to capture the essence of this transition—the two-dimensional Hubbard model—has thus far defied a general solution by any analytical or numerical method, even though tremendous progress has been made in elucidating its phase behaviour[49].

The interplay between short-range and long-range interactions near the metal–insulator transition generically yields modulated structures with 'intertwined' order parameters and a multitude of spontaneously broken rotational and translational symmetries[50] (Fig. 1). The resulting complex spin and charge textures resemble partially ordered structures in classical liquid crystals, but the winding of the quantum phase across these textures gives rise to novel phenomena without analogues in classical physics. For instance, the phase-coherent superposition of superconductivity and charge order ('pair density wave') can engender spontaneously decoupled superconducting sheets in three-dimensional materials (recently observed in underdoped cuprate superconductors[51]) as well as unconventional vortex excitations (a prediction that is yet to be confirmed). Quantum phase transitions between modulated structures driven by doping, pressure and external electric and magnetic fields can profoundly affect the thermodynamic and transport properties, as illustrated by recent experiments on high-temperature superconductors (Fig. 4). Understanding and controlling the influence of doping-induced disorder and lattice strain on these transitions, and on the mesoscale properties of correlated-electron systems in general[52], remains an important challenge for quantum materials research—not least because such electronically inhomogeneous states provide fertile ground for various applications by virtue of their high susceptibility to external parameters (see the other reviews in this issue).

Yet the discovery of high-temperature superconductivity has shown that beautifully robust, homogeneous many-electron states can emerge out of the messy soup of strongly correlated electrons. Much progress has been made since then in explaining unconventional superconductivity and other ordering phenomena in the copper oxides[53], as well as related states in ruthenates, heavy-fermion intermetallics, organic conductors, and other correlated-electron

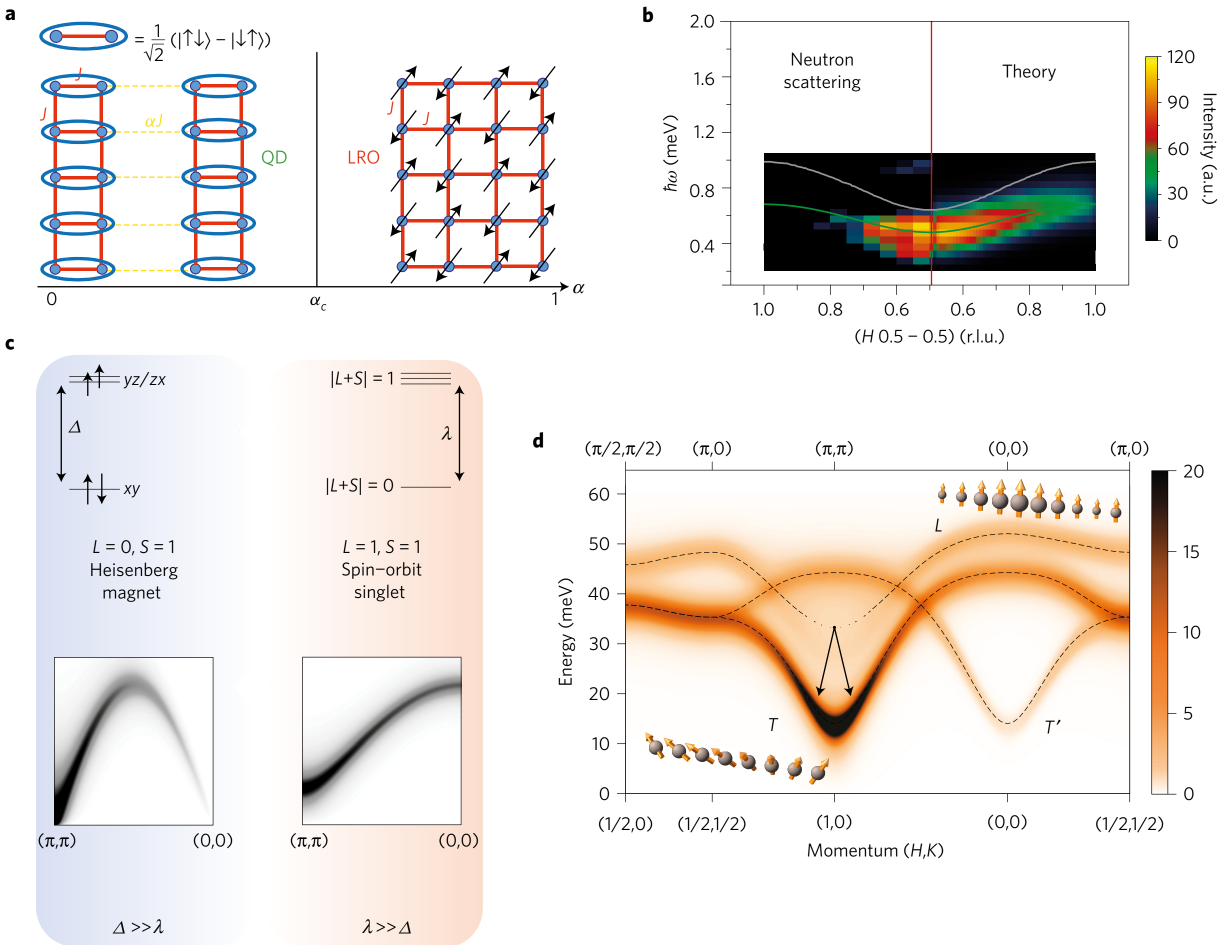

Figure 3 | Neutron scattering from Higgs modes. a, Quantum phase transition in a two-leg spin-1/2 ladder compound, driven by the strength of the inter-ladder coupling α. At the quantum-critical point ($\alpha = \alpha_c$), the ground state changes from a quantum disordered (QD) state composed of spin-singlets (left) to a long-range ordered (LRO) antiferromagnet (right), and the entanglement between single-electron states becomes long-ranged. **b**, Inelastic neutron scattering data on the spin-1/2 ladder compound $C_9H_{18}N_2CuBr_4$ as a function of crystal momentum and energy (left panel), and model calculations that attribute the dispersive excitation to a longitudinal ('Higgs') mode (right panel). **c**, Competing ground states of a spin-1 magnet in a nearly cubic lattice structure with three electronic orbitals of symmetry *xy*, *xz* and *yz*. For large crystal field splitting Δ, the orbital angular momentum is quenched, and the spin system is described by a Heisenberg Hamiltonian with an antiferromagnetic ground state (left). For large intra-atomic spin–orbit coupling λ, the ground state is a non-magnetic spin–orbit singlet (right). A quantum phase transition between both ground states occurs as a function of λ/Δ. The relative influence of λ and Δ can be inferred from the dispersion of the magnetic excitations that are displayed as a function of momentum and energy. **d**, Representation of inelastic neutron scattering data on the spin-1 antiferromagnet Ca_2RuO_4 together with the results of model calculations (dashed lines) that confirm the dominant influence of spin-orbit coupling. The high-energy mode can be assigned to a Higgs mode which is stable over much of the Brillouin zone but decays into transverse spin waves near the antiferromagnetic ordering vector. Adapted from ref. 28, Macmillan Publishers Ltd (**a**,**b**); and ref. 27, Macmillan Publishers Ltd (**c**,**d**).

materials[54]. In particular, multiple lines of evidence akin to those in the iron-based superconductors have demonstrated a key role of collective electronic modes—especially spin fluctuations—in driving these transitions[55]. However, fundamental questions about the nature of the collective dynamics remain at the frontier of current research. Do they predominantly entangle electrons on nearest-neighbour lattice sites, or is the long-range entanglement generically associated with quantum phase transitions between different forms of collective order[56] (Fig. 3) of crucial importance? And how do they affect the anomalous normal-state properties, including the enigmatic temperature-linear resistivity that has been observed in a diverse set of quantum materials? These and other questions are now being addressed with innovative experimental tools (including transport and thermodynamics in magnetic fields up to 100 T (Fig. 4)[57–59], novel spectroscopies such as resonant inelastic X-ray scattering[60], and pump–probe methods that generate coherent states far from equilibrium[61]) and clever computational schemes (including machine-learning algorithms[62] as well as *ab initio* methods that provide direct insight into the structure of the many-electron wavefunction[63]). Quantum simulators based on cold-atom systems are poised to generate additional insight into the many-electron problem[64,65]. With any luck, this concerted effort will soon enable the controlled manipulation of strongly correlated electron systems, and eventually the theory-guided design[66] of quantum materials with collective electronic order sufficiently robust to withstand thermal decoherence at room temperature and beyond.

Novel phenomena due to geometry of electron wavefunctions

The simplest topological phases, which are based on the geometry of single-electron wavefunctions, have seen a tremendous renaissance in the past decade. The intellectual ancestor of these phases is the integer quantum Hall effect (IQHE) discovered in 1980: two-dimensional electron systems in strong magnetic fields can show incredibly precise quantization in their transport properties[67]. It

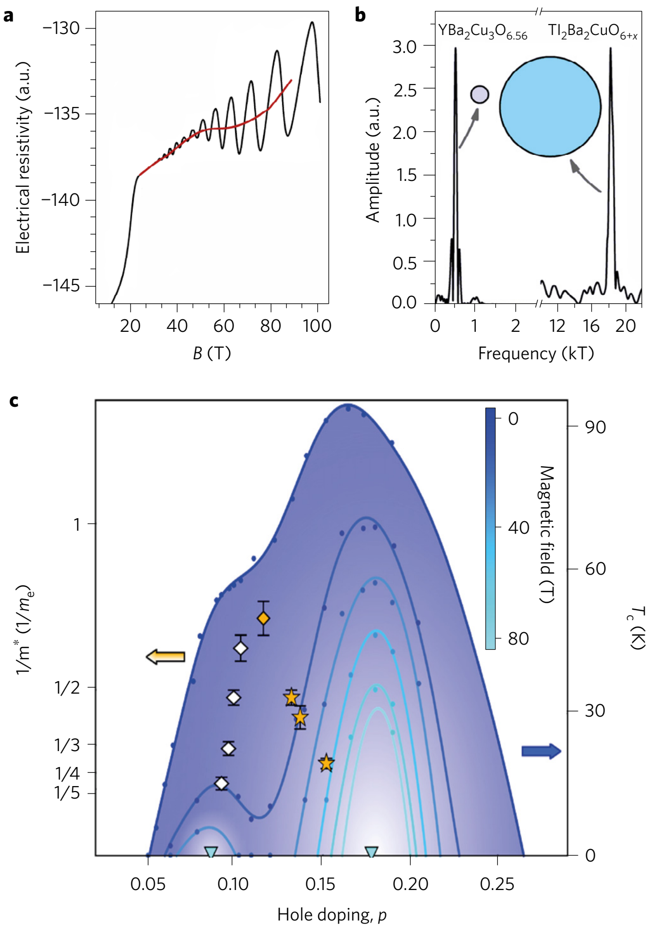

Figure 4 | Quantum oscillations and quantum criticality in high-temperature superconductors. a, Electrical resistivity of $YBa_2Cu_3O_{6.6}$ as a function of magnetic field. A field of ~20 T weakens superconductivity sufficiently to reveal quantum ('Shubnikov de Haas') oscillations whose frequency reflects the dimensions of the Fermi surface. **b**, Fourier transform of the quantum oscillations in underdoped (left) and overdoped (right) cuprates—that is, materials with doping levels below and above those required for the maximal superconducting transition temperature T_c. The size of the Fermi surface inferred from these measurements changes abruptly around optimal doping, indicating a quantum phase transition. **c**, Doping dependence of T_c (right axis) in $YBa_2Cu_3O_{6+x}$ in different magnetic fields (indicated by the colour scale). The left axis indicates the inverse effective mass of the charge carriers inferred from the quantum oscillations. The incipient divergence of the effective mass around optimal doping provides additional indications of a quantum phase transition. The relationships between this transition, the mechanism of high-T_c superconductivity, and the 'strange metal' properties of the cuprates above T_c are subjects of intense investigation in quantum materials research. Adapted from ref. 58, Macmillan Publishers Ltd (**a**,**b**); and ref. 59, AAAS (**c**).

turns out that in addition to the standard textbook picture of the IQHE, which is based on Landau levels (the eigenstates of an electron moving in a magnetic field in free space), another picture was developed around the same time by Thouless, Kohmoto, Nightingale and den Nijs (TKNN) that is more directly relevant to an electron moving in a crystal[68].

This work explained that the integer quantum Hall effect is associated with nontrivial topological properties (Chern numbers or TKNN integers) of the Bloch states of the electrons in a crystal. A few years later, Haldane gave an example of how the integer quantum Hall effect could arise in a simple model of a crystal with zero magnetic field on average[69]. The work of Haldane and Thouless was recognized by the 2016 Nobel Prize in Physics, and turns out to be crucial for many of the more recent developments in this area.

Topological insulators. Nontrivial electronic topology can be generated by spin–orbit coupling[70] instead of a magnetic field, leading to 'topological insulators'[71]. Because spin–orbit coupling respects time-reversal symmetry, unlike a magnetic field, there are some key differences. There is a two-dimensional topological insulator phase, which has edge states similar to those of the quantum Hall effect, and in some limits these support a quantized spin Hall effect. A difference is that the edge states are protected from backscattering by time-reversal symmetry rather than by an energy gap as in the quantum Hall case. This state was found in transport experiments in HgTe quantum wells[72] following theoretical predictions[73].

The quantum Hall effect is fundamentally a two-dimensional state, although there might be materials ('Chern insulators') that could be viewed as stacks of quantum Hall layers. However, there does exist a three-dimensional topological insulator phase in bulk materials[74–76]. Its mathematical connection to the two-dimensional case is fairly complicated, but the physical consequences are similar: there are metallic surface states, and in the simplest case (realized in Bi_2Se_3 (ref. 77), Bi_2Te_3 (ref. 78), and other more complicated materials) the surface contains a single 'Dirac cone' of electrons, similar to the cones in the electronic structure of graphene displayed in Fig. 5. The 3D topological insulator supports a quantized magnetoelectric effect[79,80], which may have been observed in recent terahertz optical experiments[81]. Magnetically doping a 3D topological insulator produces a quantum anomalous Hall state[82], which is similar to the IQHE except that orbital magnetic fields are believed to be less important than spin–orbit coupling in driving the effect.

Topological semimetals. Recent years have seen a great growth of interest in topological semimetals, which embody two different ways of generalizing the effectively massless electrons of graphene (a two-dimensional semimetal) to bulk materials. Dirac's original equation for a massless fermion in three dimensions, when interpreted in a solid-state context, describes a point where four bands touch and as a result the effective Bloch Hamiltonian consists of four by four matrices, similar to those in the 3D Dirac equation. The first examples of such Dirac semimetals (the material is a semimetal with a pointlike Fermi surface if the Fermi energy passes only through Dirac points) were discovered just a few years ago[83–85], and the search for additional examples continues; of course, Dirac quasiparticles famously arise in superconductors near nodes of the gap function (Fig. 5).

Soon after Dirac's celebrated equation was first written down, Hermann Weyl pointed out that a massless particle can have a fixed 'handedness', allowing the Dirac equation to be broken into two-component halves. (A different way of separating the Dirac equation, due to Majorana, is mentioned below.) These two-band degenerate points can be realized in materials that break either inversion or time-reversal symmetry. A recent realization is that Weyl points have a topological meaning and can appear in phase transitions between normal and topological insulators[86,87]: each Weyl point can be assigned a topological number, directly related to the Chern numbers described above in the IQHE, and the total number of Weyl points in a material is always zero, as explained by the Nielsen–Ninomiya theorem from particle physics[88].

It is worth pointing out that, while the theoretical discussion of Weyl semimetals goes back to the 1930s, the new topological tools of theorists were crucial in their experimental discovery: there are Fermi arc surface states[89], and it was the detection of these Fermi arcs in photoemission that gave strong proof that the phase had indeed been found. Dirac and, particularly, Weyl semimetals should support several transport and optical phenomena that are

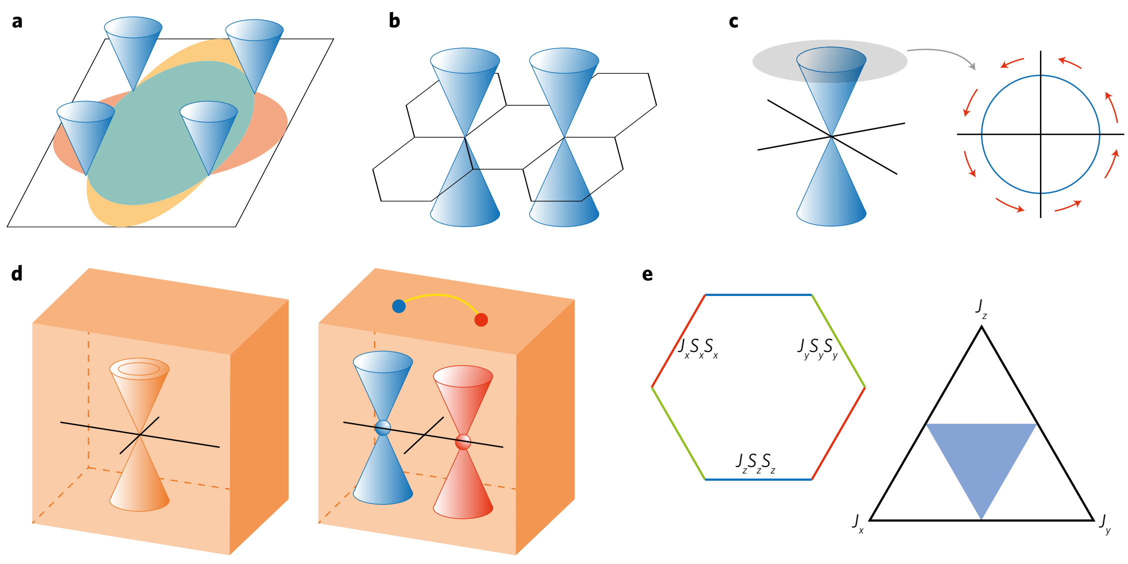

Figure 5 | Examples of massless fermions in quantum materials. A measure of the rapid progress in correlated and topological materials is the understanding of different mechanisms that generate massless fermionic excitations. In **a**–**d**, black lines indicate momentum-space directions, energy is vertical. **a**, The Bogoliubov quasiparticles of a *d*-wave superconductor are superpositions of electron and hole that can be created with arbitrarily low energies above the ground state (Fig. 1g) at certain momenta. **b**, The band structure of graphene contains massless Dirac electrons at two points in the Brillouin zone. **c**, Topological insulators are three-dimensional materials that intrinsically support massless electrons at their surfaces. The spin direction of a surface electron (marked in red) is determined by ('locked to') its momentum. **d**, 3D Dirac (left) and Weyl (right) semimetals are both generalizations of the Dirac cones in graphene; the difference is that in the Weyl case, the breaking of either inversion or time-reversal symmetry allows four-fold degenerate Dirac cones to split into two-fold degenerate Weyl cones, each of which has a topological integer 'charge' (marked in red and blue). Surface electrons in Weyl semimetals exhibit a line of zero-energy states ('Fermi arc') in momentum space. **e**, A long-sought state of frustrated magnets that may already have been discovered has gapless fermionic excitations above the ground state. These are different from the bosonic magnons (quanta of spin waves) that exist in more conventional magnets. One model supporting such excitations is the spin-anisotropic Kitaev honeycomb lattice Hamiltonian (left), where only one spin component is active on each set of bonds (*x*, *y*, *z*). Its phase diagram with gapped (white) and gapless (blue) excitations is shown on the right.

still being sought in experiments[90–92]. These phenomena in many cases are related to the 'chiral anomaly' beloved of particle physicists. A single Weyl fermion is anomalous in that its quantum theory, when coupled to electromagnetism, does not conserve charge; there are subtle implications of this in solids, where there are always multiple Weyl points (so that the total charge cancels) but they may occur at different energies.

There are several theoretically possible phenomena, such as quantized nonlinear optical properties, that will become observable if new materials can be found: the most studied Weyl semimetals belong to the TaAs family[93–95] and have many Weyl points plus additional symmetries that forbid some of the most exciting predicted effects. Similarly, better materials would enable improved observation of the Fermi arc surface states by means other than photoemission. Finally, Dirac and Weyl semimetals, just like topological insulators, may serve as independent-electron starting points for correlated phenomena such as superconductivity[96] and fractionalization, and to these we now turn. It should be noted that even states that do not intrinsically require electron–electron interactions might be helped to form by such interactions, as in the possible topological Kondo insulator[97] SmB_6. Also, considerable recent progress has happened at the intersection of crystalline symmetry with topology—for example, in the discovery of surface states in the topological crystalline insulator SnTe[98].

Interplay of many-body physics and topology

Only a few years after the experimental discovery of the integer quantum Hall effect, an even more unusual state of electrons was discovered that also exists in two-dimensional electron gases in a strong magnetic field[99]. The fractional quantum Hall effect (FQHE), unlike the IQHE, cannot be understood in terms of nearly independent electrons. Instead, strong Coulomb repulsion creates an incompressible quantum liquid with remarkable properties. Its fundamental excitations have fractional charge: for example, an electron added to the FQHE state at 1/3 filling (that is, when only 1/3 of the empty orbitals in the lowest Landau level are occupied) breaks up into three quasiparticles, each of charge $e/3$ (ref. 100).

Another unexpected feature of quasiparticles in this complex liquid is fractional statistics: they have statistics that is neither bosonic nor fermionic. Quantum mechanics textbooks explain the two types of statistics of point particles in three spatial dimensions as being classified by even (bosonic) or odd (fermionic) representations of the permutation group. Prescient theoretical work pointed out in 1976 that strictly two-dimensional particles could have many other types of statistics[101]. Statistics can be observed by exchanging the locations of particles, but in two dimensions the path by which that exchange happened (the 'braiding') becomes relevant, and statistics are classified by representations of the braid group rather than of the permutation group.

We will say a bit more about exotic statistics because they have become a major driver of research into the FQHE and, in recent years, the topological superconductors mentioned below. The braid group is a much more complicated beast than the permutation group. FQHE states have been found that are believed to realize both 'Abelian' and 'non-Abelian' statistics. The term Abelian just means commutative, and in a state with Abelian statistics, such as the FQHE state at filling 1/3 mentioned above, each braiding process of the quasiparticles gives a phase factor, not necessarily ± 1, that acts on the (non-degenerate) state.

That might well seem complicated enough, but in FQHE samples of very high quality, new plateaus emerged that could not be understood with any known Abelian state[102]. Theorists had written down wavefunctions that supported non-Abelian statistics[103]: there is a degenerate subspace of states when the system has many

NATURE PHYSICS DOI: 10.1038/NPHYS4302

quasiparticles, and a braiding of the quasiparticles acts as a unitary matrix on this subspace. In some FQHE states, there are so many possible matrices for different braidings that braiding quasiparticles can in principle serve as a universal quantum computer[104].

Topological superconductivity. This area remained somewhat abstruse until it was realized that even a simple superconducting wavefunction, of the type considered in the classic theory of Bardeen, Cooper and Schrieffer[3], could also support non-Abelian statistics[105]. If the superconductor has the right pairing symmetry (for example, $p_x + ip_y$) then ordinary vortex cores actually trap Majorana zero modes with non-Abelian statistics. (These zero modes bear surprisingly little relation to the Majorana fermions originally discussed as fundamental particles in the early days of quantum mechanics, which have ordinary fermionic statistics.) The search for Majorana modes is mostly taking place at the moment not in FQHE states but in what is normally meant by 'materials': we would like very much to find a solid material that by itself superconducts and supports Majorana modes. Sr_2RuO_4 is an example of a material that has been actively discussed in this context[106]. Such a material would, aside from its fundamental interest by virtue of non-Abelian statistics, offer a new path to quantum computation.

Another route is to combine relatively straightforward materials into a heterojunction or other structure that supports Majorana zero modes. One material is a standard *s*-wave superconductor, and the other material can be either a topological insulator or a semiconductor with strong spin–orbit coupling in a magnetic field. Either way, the key feature of the non-superconducting part of the heterojunction is that it only has a single Fermi surface sheet, even including spin.

There are a number of promising experimental results indicating zero-bias peaks in tunnelling[107–109] that could be from Majorana zero modes, but not yet any demonstration of braiding. An 'existence proof' of topological superconducting states is provided by one phase of superfluid helium, which can be understood as a topological superfluid of neutral atoms, complete with zero modes and surface states[110]. We now turn to how fractional quasiparticles can exist in other kinds of materials, including those with no mobile electrons.

Spin liquids. One can ask what made the partially filled Landau level a fertile ground for the occurrence of exotic correlated FQHE states. Part of the answer is that a partly filled Landau level has a very large degeneracy if only kinetic energy is concerned, which means that there is no energy barrier for interactions to overcome. Instead, interactions form complicated superpositions of the degenerate noninteracting states.

Another useful type of degeneracy in solids occurs as a consequence of magnetic frustration. A frustrated magnet is one in which not all of the magnetic interaction energies can be simultaneously minimized, and many of these have degenerate ground states at least for simplified models of the interaction. Additional interactions, or residual parts of the original interactions, can again form superpositions of the originally degenerate spin configurations. This mechanism leads in some models of frustrated magnetism to quantum spin liquids that include topological order similar to that in FQHE states and have fractional excitations with half-integer spin rather than the integer spin of conventional magnons (spin waves).

The search for quantum spin liquids has been a major goal of the frustrated magnetism community for many years, and several types are now understood. Chiral spin liquids are closely related to simple FQHE states. Another type of spin liquid, called a Z_2 spin liquid, appears in quantum dimer models[111], which are intended to capture the singlet correlations relevant to cuprate superconductors, and a spin liquid of this type may appear in the nearest-neighbour Heisenberg model on the kagome lattice as possibly realized in herbertsmithite[112].

Perhaps most intriguing and mysterious are gapless spin liquids. Topological order can be defined rigorously in states with an energy gap above the subspace of ground states. The type of order in a gapless spin liquid is harder to define, but there are explicit examples of models with gapless spin liquid ground states. A famous example on the honeycomb lattice was introduced by Kitaev[113] (Fig. 5) and has been used as a basis for understanding inelastic neutron scattering on $RuCl_3$ (ref. 114). Even the kagome lattice antiferromagnet may actually be a gapless state according to the most recent numerics, which shows how even simple Hamiltonians can host competing topological orders of various types.

Outlook

The convergence of many-body physics and topology is now driving several frontiers of research on quantum materials. One such frontier is the electronic structure of strongly correlated metals on the verge of Mott localization. Theoretical research indicates that 'doped Mott insulators', where a small number of mobile charge carriers propagate through a slowly fluctuating background texture of correlated spins, share many of their topological properties with insulating spin liquids and are topologically distinct from the Fermi liquid that emerges for large carrier concentrations far away from the Mott insulator[115,116]. At the quantum transition separating these phases, the many-electron wavefunction is entangled over macroscopic distances, and novel fractionalized quasiparticles unlike those on either side of the transition have been predicted[117]. To establish whether these phases and phase transitions are realized in the copper oxides and other doped Mott insulators, and to what extent they influence the anomalous physical properties of these materials, it will be crucial to devise direct experimental tests of the topological properties of correlated metals analogous to those that have been reported for simple insulators and semiconductors. The rapidly evolving capability to prepare complex *d*- and *f*-electron materials in confined geometries[118] will be a powerful resource for experimental creativity.

Progress is also happening in theoretical methods. The concepts of wavefunction geometry and topology that helped identify new topological phases are now being used to analyse a wide variety of phenomena and materials. Even properties such as polarization, magnetoelectricity and photocurrent that are not thought of as topological turn out to reflect Berry phases and other geometrical concepts. Entanglement in electron wavefunctions is directly related to the success of one class of numerical methods ('tensor networks') for quantum materials. Some entangled states with special structure, such as Slater determinants of free fermions, are computationally easy, but in complex states where simpler methods fail, the amount of entanglement is a key determinant of computational difficulty. The competing orders in complex materials such as cuprate superconductors remain difficult, as do topological states with strong correlations, and perhaps these are ideal 'grand challenge' problems for future exascale or quantum computers.

Quantum topology and many-electron physics also meet at the frontier of research on 'spintronic' devices that operate with spin (rather than charge) currents[119]. Both spin-polarized topological edge currents and spin supercurrents carried by quantum condensates of magnons[120] offer tantalizing perspectives of data processing with minimal dissipation. In a parallel development, skyrmions—topological defects in complex magnetic ground states generated by electronic correlations—are being explored as a novel platform for data storage and spintronics[121]. Integrating these elements into electronic devices will require a quantum materials research and development effort comparable to the one that paved the way for today's semiconductor technology. Given the multitude of innovative ideas, methods and materials that are currently being pursued, it seems safe to predict that electronic devices a generation from now will take advantage of quantum entanglement and topology at a much deeper level than today. And in the spirit of discovery that has fuelled this field of research, more surprises are guaranteed in the quest to understand and control quantum correlations in materials.

Received 3 July 2017; accepted 25 September 2017; published online 30 October 2017

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Acknowledgements

B.K. acknowledges financial support by the European Research Council under Advanced Grant No. 669550 (Com4Com) and by the German Science Foundation (DFG) in the Collaborative Research Center TRR80. J.E.M. was supported by the Office of Basic Energy Sciences, Materials Sciences and Engineering Division, of the US Department of Energy under Contract No. DE-AC02-05CH11231, and the National Science Foundation under grant DMR-1507141.

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Competing financial interests

The authors declare no competing financial interests.