If Neutrinos are Tachyons, we may explain Dark Energy and Dark Matter

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Start with the OPERA experiment of 2011. 20 GeV neutrinos appeared to travel faster than light by about 1/40,000.

From the formula $E = mc^2/\sqrt{v^2/c^2 - 1}$,

$$(v-c)(v+c)/c^2 = (mc^2/E)^2. \qquad (1)$$

So that would imply a tachyon neutrino with a mass of about 100 MeV. But we are pretty sure that neutrino mass is around 0.1 eV. So the OPERA result going away tells us nothing about possible neutrino tachyons of such a low mass. That is what we are considering.

Classical particles are described by a world-line $\xi^\mu(\tau)$ with a Lorentz invariant form that leads us to note three categories:

$$\eta_{\mu\nu}\,\dot{\xi}^\mu\,\dot{\xi}^\nu = \epsilon \tag{2}$$

$$\begin{aligned} ordinary\,particles\;(v < c): &\qquad \epsilon = +1 \\ massless\,particles\;(v = c): &\qquad \epsilon = 0 \\ tachyons\;(v > c): &\qquad \epsilon = -1 \end{aligned}$$

We usually define the 4-vector $p^\mu = m\dot{\xi}^\mu$. For tachyons this is a space-like vector and so one asks, What about negative energy states?

Let's look at a space-time diagram for a general interaction process. Figure 1 shows four particles involved in the reaction, $n \rightarrow p + e + \nu$, where I imagine that the neutrino is a tachyon.

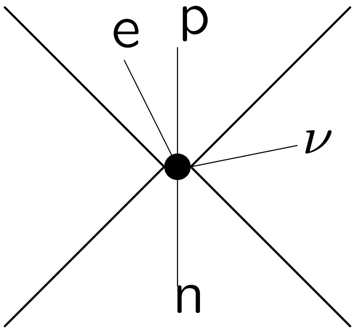

Figure 1. Reaction with an outgoing tachyon.

Now look at Figure 2. Is this a picture of the reaction $n \to p + e + \nu$ with the neutrino carrying off negative energy; or is this a picture of the reaction $n + \nu \to p + e$ with positive energy for all participants?

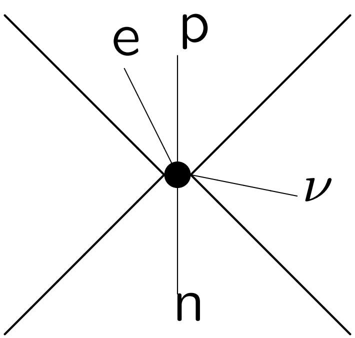

Figure 2. Reaction with an incoming tachyon.

This ”problem” is the same as noting that the mass-shell equation,

$$p^{\mu}\, p_{\mu} = E^2 - p^2 = \pm m^2, \tag{3}$$

gives us two separate hyperboloids for ordinary particles (plus sign) but a single hyperboloid for tachyons (minus sign). For ordinary particles, we manage to reinterpret the negative energy solutions as antiparticles and give them positive energy. For tachyons we need to see what it means when we look at a positive energy particle from a different Lorentz frame, where it appears to have negative energy.

Look again at the space-time diagrams above. A ''positive energy'' tachyon will have $dt/d\tau >$ 0 and its trajectory will be seen moving upward - as in Figure 1 - and so we would call that an outward going particle if it sits above the interaction region in time; and we would call it an inward moving particle if the trajectory sits below the interaction region - as in Figure 2. But from another reference frame we may have $dt/d\tau < 0$ for what was formerly an outgoing particle and so this will now look like an incoming particle.

The lesson is that *the labels "in" and "out" are Lorentz invariant for ordinary particles but NOT for tachyons.* Does this matter? No. The physical law which we call the conservation of total energy and momentum is written,

$$\sum_{out} p_j^\mu - \sum_{in} p_i^\mu = 0. \tag{4}$$

This is true in any Lorentz frame, even though the individual terms in this equation each transform. In going from Figure 1 to Figure 2 we need only move one p^μ from the "out" summation to the "in" summation.

Now we look at tachyons in General Relativity.

For ordinary particles we write the Energy-Momentum tensor as,

$$T^{\mu\nu}(x) = m \int d\tau\, \dot{\xi}^{\mu}\, \dot{\xi}^{\nu}\, \delta^4(x - \xi(\tau)), \qquad (5)$$

and, at first, I used this same formula for tachyons.

For a free particle we have the familiar representation,

$$\xi^{\mu}(\tau) = (\gamma\tau, \gamma\mathbf{v}\,\tau), \qquad \gamma = 1/\sqrt{|1 - v^2/c^2|}. \qquad (6)$$

Thus, for a very low energy ordinary particle ($v << c$) the dominant term is

$$T^{00}(x) = m\delta^3(\mathbf{x} - \mathbf{v}t). \qquad (7)$$

and for a very low energy tachyon ($v >> c$) the dominant terms are

$$T^{ij}(x) = m\gamma v_i \, v_j \, \delta^3(\mathbf{x} - \mathbf{v}t). \qquad (8)$$

These space-components of $T^{\mu\nu}$ can be very large; and this leads to interesting cosmological modeling.

In my 2011 paper I showed that this leads to attractive forces among co-linear flows of tachyons; and I predicted the possibility of such tachyon ”ropes” becoming localized, say, within a galaxy, and creating strong local gravitational fields that could produce the observational effects now ascribed to Dark Matter.

Then, last year, I was led to revise that theory, as follows.

Let's derive that formula for tachyons' energy-momentum tensor from some general principle. General Relativity may be constructed from an action principle with a Lagrangian density that looks like this.

$$\mathcal{L} = \sum m \int d\tau \, \sqrt{\epsilon g_{\mu\nu}(x)\dot{\xi}^{\mu} \, \dot{\xi}^{\nu}}\delta^4(x - \xi(\tau)) + \frac{\sqrt{|detg|}}{8\pi G} \, R \quad ,(9)$$

where we sum over all the particles and R is the scalar form of the Riemann curvature tensor, which depends on the metric tensor $g_{\mu\nu}(x)$. Notice that I put an epsilon under the square root to make sure it would always be real, whether we have an ordinary particle or a tachyon.

GR textbooks show how, under variation of the metric $g_{\mu\nu}$, we get exactly Einstein's equation, with the energy momentum tensor, as we wrote it earlier, on the right hand side. Note, however, that the factor epsilon should be sitting there:

$$T^{\mu\nu}(x) = \epsilon m \int d\tau \, \dot{\xi}^{\mu} \, \dot{\xi}^{\nu} \, \delta^4(x - \xi(\tau)). \quad (10)$$

This new minus sign leads to the unconventional result of repulsive gravitational forces; and now we predict that tachyons, if they exist in the cosmos, would form a pervasive gas and contribute a *negative pressure* when we consider the Robertson-Walker model for the universe. The formula for this pressure is simply,

$$p = -m\gamma \frac{v^2}{3} \rho, \tag{11}$$

where ρ is the density of those tachyons, v is their velocity, and $\gamma = 1/\sqrt{v^2/c^2 - 1}$.

Using standard numbers for the density and energy of the Cosmic Neutrino Background, assuming that they could be tachyons with a mass around 0.1 eV/c^2, gives a numerical value for **this negative pressure that is within a factor of two of explaining what is commonly called Dark Energy.**

Thus far, considering cosmic neutrinos as tachyons in General Relativity:

ACT I - this may explain Dark Matter

alternatively,

ACT II - this may explain Dark Energy

... waiting for Act III

Let's look at a quantized field theory for a Dirac tachyon.

$$\psi(x) = \int d\omega \int d^2\hat{k}\; k^2 b_h(\omega,\hat{k})\; e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} v_h(\omega,\hat{k}) \tag{12}$$

where $k = \sqrt{\omega^2+m^2}$, $\mathbf{k} = k\;\hat{k}$, and v_h is a Dirac spinor for a tachyon (put "im" instead of "m" in the Dirac wave equation), with helicity h. The operator algebra is

$$[b_h(\omega,\hat{k}), b^\dagger_{h'}(\omega',\hat{k}')]_+ = \delta(\omega-\omega')\delta^2(\hat{k}-\hat{k}')/k^2\;\delta_{h,h'}. \tag{13}$$

and this gives us the two-point anti-commutator for the fields,

$$[\psi(x), \psi^{\dagger}(x')]_{+} = 0, \quad if \quad |t - t'| > |\mathbf{x} - \mathbf{x}'|. \tag{14}$$

This is the strong condition of causality for tachyon fields: *No signal can travel slower than the speed of light.*

Next, we look at one-particle states and then examine how to calculate the energy-momentum tensor. What we note is that the helicity h serves to signify what we call the particle and what we call the anti-particle. (For ordinary particles this job was given to the sign of the frequency.) Working from the vacuum state $|0>$ we define:

$$b_{h=+1}(\omega,\widehat{k})|0>= 0, \quad b^{\dagger}_{h=-1}(\omega,\widehat{k})|0>= 0. \tag{15}$$

Then construct one-particle states,

$$|\omega,\widehat{k},h=+1>= \sqrt{k}\; b^{\dagger}_{h=+1}(\omega,\widehat{k})|0>, \tag{16}$$

$$|\omega,\widehat{k},h=-1>= \sqrt{k}\; b_{h=-1}(\omega,\widehat{k})|0> . \tag{17}$$

The conserved energy-momentum tensor for the tachyon Dirac field is,

$$T^{\mu\nu}(x) = (i/4)\psi^{\dagger}(x)\gamma^0\gamma^5(\gamma^{\mu} \overset{\leftrightarrow}{\partial}^{\nu} + \gamma^{\nu} \overset{\leftrightarrow}{\partial}^{\mu})\psi(x), \tag{18}$$

where $\overset{\leftrightarrow}{\partial} = \overset{\rightarrow}{\partial} - \overset{\leftarrow}{\partial}$.

I want to put this operator between one-particle states; but first there are two important steps. First we embrace this operator with the notation : ... : which means "normal order", to guarantee that this will have zero expectation value in the vacuum state.

The second step is to acknowledge that we need a specific indefinite metric when taking matrix elements between one-particle states of spin one-half tachyons. This comes from investigating the "Little Group" $O(2,1)$ appropriate for tachyons in building a unitary representation of the Lorentz group. This indefinite metric H is simply the helicity operator.

The result of this calculation is,

$$< \omega, \widehat{k}, h | H : T^{\mu\nu} : | \omega, \widehat{k}, h >= h\, k^{\mu}\, k^{\nu}\ . \quad (19)$$

This says that particle and anti-particle contribute to the energy-momentum tensor with *opposite* signs. **Thus we may explain both Dark Matter and Dark Energy with the proposition that Cosmic Background Neutrinos are tachyons, with a mass in the neighborhood of** $0.1\ eV/c^2$**.**

More Work that Needs to be Done

Questions about previous results/claims

* Should chirality, rather than helicity, define particle vs anti-particle for tachyons?

* Is $k^2\, d\omega\, d^2\hat{k}$ the correct density of states formula for tachyons?

* Can we show that gravity works to localize tachyon streams?

New areas to be explored

* Fitting of tachyon neutrinos into the Standard Model of particles.

* Mass mixing for tachyon neutrinos

* Revise Standard Cosmology Theory for tachyon neutrinos (many sub-topics)

* New experiments to detect low energy tachyon neutrinos

Summary of Cosmology results - Scaling of Energy and Pressure Components

Radiation: $\rho \sim a^{-4}, \quad p \sim a^{-4}$

Cold Matter: $\rho \sim a^{-3}, \quad p \sim 0$

Cosmo Const: $\rho \sim a^{0}, \quad p \sim a^{0}$

Cold Tachyon Neutrinos:
$\rho \sim 0, \quad p \sim a^{-1.5}$ or $p \sim a^{-2}$

Something we can check now?

If low energy tachyon neutrinos cause Dark Matter effects, those effects should be lessened in the past when their Temperature was near or above 0.1 eV. Is there data on Dark Matter effects as a function of z?

For slow particles under Newton's Gravity,

$$H = \sum_a \frac{1}{2} m_a v_a^2 - \sum_{a<b} \frac{G m_a m_b}{|\mathbf{x}_a - \mathbf{x}_b|}. \qquad (20)$$

And from Einstein's GR we now get,

$$H = -\sum_a \omega_a m_a \gamma_a - \sum_{a,b} \frac{G(\omega_a m_a \gamma_a)(\omega_b m_b \gamma_b)}{|\mathbf{x}_a - \mathbf{x}_b|} Z_{ab},$$

$$Z_{ab} = 2 - 4\mathbf{v}_a \cdot \mathbf{v}_b + (v_a^2 + v_b^2) - (\epsilon_a \gamma_a^2 + \epsilon_b \gamma_b^2 + 1) \times$$
$$\times [(1 - \mathbf{v}_a \cdot \mathbf{v}_b)^2 - \frac{1}{2}(1 - v_a^2)(1 - v_b^2)].$$

For a static system, including tachyons.

arXiv:1902.03621 [gr-qc]

”For every complex problem there is a simple solution. And it’s always wrong.”

–H. L. Mencken